Lecture Notes in Mathematics 1943

FONDAZIONE CIME ROBERTO CONTI

CENTRO INTERNAZIONALE MATEMATICO ESTIVO
INTERNATIONAL MATHEMATICAL SUMMER CENTER

C.I.M.E. means Centro Internazionale Matematico Estivo, that is, International Mathematical Summer Center. Conceived in the early fifties, it was born in 1954 and made welcome by the world mathematical community where it remains in good health and spirit. Many mathematicians from all over the world have been involved in a way or another in C.I.M.E.'s activities during the past years.

So they already know what the C.I.M.E. is all about. For the benefit of future potential users and co-operators the main purposes and the functioning of the Centre may be summarized as follows: every year, during the summer, Sessions (three or four as a rule) on different themes from pure and applied mathematics are offered by application to mathematicians from all countries. Each session is generally based on three or four main courses (24–30 hours over a period of 6-8 working days) held from specialists of international renown, plus a certain number of seminars.

A C.I.M.E. Session, therefore, is neither a Symposium, nor just a School, but maybe a blend of both. The aim is that of bringing to the attention of younger researchers the origins, later developments, and perspectives of some branch of live mathematics.

The topics of the courses are generally of international resonance and the participation of the courses cover the expertise of different countries and continents. Such combination, gave an excellent opportunity to young participants to be acquainted with the most advance research in the topics of the courses and the possibility of an interchange with the world famous specialists. The full immersion atmosphere of the courses and the daily exchange among participants are a first building brick in the edifice of international collaboration in mathematical research.

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Luis L. Bonilla (Ed.)

Inverse Problems and Imaging

Lectures given at the
C.I.M.E. Summer School
held in Martina Franca, Italy
September 15–21, 2002

Chapters by:

A. Carpio · O. Dorn · M. Moscoso · F. Natterer
G.C. Papanicolaou · M.L. Rapún · A. Teta

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Preface

Nowadays, we are facing numerous important imaging problems, as for example, the detection of anti-personal land mines in post-war remediation areas, detection of unexploded ordnances (UXO), nondestructive testing of materials, monitoring of industrial processes, enhancement of oil production by efficient reservoir characterization, and the various exciting and emerging developments in noninvasive imaging techniques for medical purposes – computerized tomography (CT), magnetic resonance imaging (MRI), positron emission tomography (PET) and ultrasound tomography, to mention only a few. It is broadly recognized that these problems can only be solved by a joint effort of experts in mathematical and technical sciences.

The CIME Summer School on Imaging, held in Martina Franca, Italy, from 15 to 21 September, 2002, encompassed the theory and applications of imaging in different disciplines including, medicine, geophysics, engineering, etc. The Summer School brought together leading experts in mathematical techniques and applications in many different fields, to present a broad and useful introduction for non-experts and practitioners alike to many aspects of this exciting field. The main lecturers were Simon Arridge, Frank Natterer, George C. Papanicolaou and William Symes. Seminars on related special topics were contributed by Oliver Dorn, Miguel Moscoso and Alessandro Teta. Among the different topics dealt with in the school, we may cite X-ray tomography, diffusive optical tomography with possible applications to tumor detection in medicine by optical means, electromagnetic induction tomography used in geophysics, and techniques of seismic tomography to image the wave velocity structure of a region and to obtain information on possible oil or gas reservoirs, etc. Furthermore, there were extensive discussions on the mathematical bases for analyzing these methods. The mathematical and computational techniques that are used for imaging have many common features and form a rapidly developing part of applied mathematics.

The present volume contains a general introduction on image reconstruction by M. Moscoso, some of the lectures and presentations given in the Summer School (F. Natterer, O. Dorn et al., M. Moscoso, G. Dell'Antonio et al.), and two additional lectures on other imaging techniques by A. Carpio and M.L. Rapún and by O. Dorn.

The lectures by Prof. Frank Natterer introduce the mathematical theory and the reconstruction algorithms of computerized X-ray tomography. These lectures give a short account of integral geometry and the Radon transform, reconstruction algorithms such as the filtered back projection algorithm, iterative methods (for example, the Kaczmarz method) and Fourier methods. They also comment on the three-dimensional case, which is the subject of current research. Many of the fundamental tools and issues of computerized tomography, such as back projection, sampling, and high frequency analysis, have their counterparts in more advanced imaging techniques for impedance, optical or ultrasound tomography and are most easily studied in the framework of computerized tomography.

The chapter by O. Dorn, H. Bertete-Aguirre and G.C. Papanicolaou reviews electromagnetic induction tomography, used to solve imaging problems in geophysical and environmental imaging applications. The focus is on realistic 3D situations which provide serious computational challenges as well as interesting novel mathematical problems to the practitioners. The chapter first introduces the reader to the mathematical formulation of the underlying inverse problem; it then describes the theory of sensitivity analysis in this application; it proposes a nonlinear reconstruction algorithm for solving such problems efficiently; it discusses a regularization technique for stabilizing the reconstruction; and finally it presents various numerical examples for illustrating the discussed concepts and ideas.

The chapter by M. Moscoso presents optical imaging of biological tissue using the polarization effects of a narrow beam of light. The biological tissue is modeled as a continuous medium which varies randomly in space and which contains inhomogeneities with no sharp boundaries. This differs from the more usual point of view in which the biological tissue is modeled as a medium containing discrete spherical particles of the same or different sizes. The propagation of light is then described by a vector radiative transport equation which is solved by a Monte Carlo method. A discussion on how to use polarization to improve image reconstruction is given as well.

The chapter by A. Carpio and M.L. Rapún explains how to use topological derivative methods to solve constrained optimization reformulations of inverse scattering problems. This chapter gives formulas to calculate the topological derivatives for the Helmholtz equation and for the equations of elastic waves. Furthermore they explain and implement a practical iterative numerical scheme to detect objects based on computing the topological derivative of a cost functional associated to these equations in successive approximate domains. Many examples of reconstruction of objects illustrate this method.

The chapter by O. Dorn deals with an inverse problem in underwater acoustic and wireless communication. He establishes a link between the time-reversal and adjoint methods for imaging and proposes a method for solving the inverse problem based on iterative time-reversal experiments.

Lastly, the chapter by G. Dell'Antonio, R. Figari and A. Teta reviews the theory of Hamiltonians with point interactions, i.e., with potentials supported on a finite set of points. This chapter studies the mathematical basis of scattering

with point scatterers, analyzes how such an idealized situation relates to short-range potentials and discusses situations in which the strength of the potential depends on the wave function as it has been proposed in the physics literature on double barriers and other nanostructures. Knowing the solution of the direct problem is of course a prerequisite to be able to image the scatterers from measurements on a boundary.

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