Lecture Notes in Mathematics 1939

FONDAZIONE CIME
ROBERTO CONTI
CENTRO INTERNAZIONALE MATEMATICO ESTIVO
INTERNATIONAL MATHEMATICAL SUMMER CENTER

C.I.M.E. means Centro Internazionale Matematico Estivo, that is, International Mathematical Summer Center. Conceived in the early fifties, it was born in 1954 and made welcome by the world mathematical community where it remains in good health and spirit. Many mathematicians from all over the world have been involved in a way or another in C.I.M.E.'s activities during the past years.

So they already know what the C.I.M.E. is all about. For the benefit of future potential users and co-operators the main purposes and the functioning of the Centre may be summarized as follows: every year, during the summer, Sessions (three or four as a rule) on different themes from pure and applied mathematics are offered by application to mathematicians from all countries. Each session is generally based on three or four main courses ($24-30$ hours over a period of 6-8 working days) held from specialists of international renown, plus a certain number of seminars.

A C.I.M.E. Session, therefore, is neither a Symposium, nor just a School, but maybe a blend of both. The aim is that of bringing to the attention of younger researchers the origins, later developments, and perspectives of some branch of live mathematics.

The topics of the courses are generally of international resonance and the participation of the courses cover the expertise of different countries and continents. Such combination, gave an excellent opportunity to young participants to be acquainted with the most advance research in the topics of the courses and the possibility of an interchange with the world famous specialists. The full immersion atmosphere of the courses and the daily exchange among participants are a first building brick in the edifice of international collaboration in mathematical research.

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Mixed Finite Elements, Compatibility Conditions, and Applications

Lectures given at the
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Preface

This volume is a collection of the notes of the C.I.M.E. course "Mixed finite elements, compatibility conditions, and applications" held in Cetraro (CS), Italy, from June 26 to July 1, 2006.

Since the early 1970s, mixed finite elements have been the object of wide and deep study by the mathematical and engineering communities. The fundamental role of mixed methods for many application fields has been recognized worldwide and their use has been introduced in several commercial codes. An important feature of mixed finite elements is the interplay between theory and application: on the one hand, many schemes used for real life simulations have been cast in a rigorous framework, and on the other, the theoretical analysis makes it possible to design new schemes or to improve existing ones, based on their mathematical properties. Indeed, due to the compatibility conditions required by the discretization spaces to provide stable schemes, simple minded approximations generally do not work and the design of suitable stabilizations gives rise to challenging mathematical problems.

The course had two main goals. The first one was to review the rigorous setting of mixed finite elements and to revisit it after more than 30 years of practice; this resulted in developing a detailed a priori and a posteriori analysis. The second one was to show some examples of possible applications of the method.

We are confident this book will serve as a basic reference for people exploring the field of mixed finite elements. This "Lecture Notes" cover the theory of mixed finite elements and applications to Stokes problem, elasticity, and electromagnetism.

Ricardo G. Durán had the responsibility of reviewing the general theory. He started with the description of the mixed approximation of second-order elliptic problems (a priori and a posteriori estimates) and then extended the theory to general mixed problems, thus leading to the famous inf–sup conditions.

The second course on Stokes problem has been given by Daniele Boffi, Franco Brezzi, and Michel Fortin. From the basic application of the inf–sup theory to the linear Stokes system, stable Stokes finite elements have been analyzed, and general stabilization techniques have been described. Finally, some results on visco-elasticity have been presented.

Richard S. Falk has dealt with the mixed finite element approximation of the elasticity problem and, more particularly, of the Reissner–Mindlin plate problem. The corresponding notes are split into two parts: in the first one, recent results linking the de Rham complex to finite element schemes have been reviewed; in the second, classical Reissner–Mindlin plate elements have been presented, together with some discussion on quadrilateral meshes.

Leszek Demkowicz has given a general introduction to the exact sequence (de Rham complex) topic, which turns out to be a fundamental tool for the construction and analysis of mixed finite elements, and for the approximation of problems arising from electromagnetism. The results presented here use special characterization of traces for vector-valued functions in Sobolev spaces.

We thank all the lecturers and, in particular, Franco Brezzi, who laid the foundation for the analysis of mixed finite elements, for his active participation in this C.I.M.E. course.

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