Lecture Notes in Mathematics 1932

FONDAZIONE CIME ROBERTO CONTI
CENTRO INTERNAZIONALE MATEMATICO ESTIVO
INTERNATIONAL MATHEMATICAL SUMMER CENTER

C.I.M.E. means Centro Internazionale Matematico Estivo, that is, International Mathematical Summer Center. Conceived in the early fifties, it was born in 1954 and made welcome by the world mathematical community where it remains in good health and spirit. Many mathematicians from all over the world have been involved in a way or another in C.I.M.E.'s activities during the past years.

So they already know what the C.I.M.E. is all about. For the benefit of future potential users and co-operators the main purposes and the functioning of the Centre may be summarized as follows: every year, during the summer, Sessions (three or four as a rule) on different themes from pure and applied mathematics are offered by application to mathematicians from all countries. Each session is generally based on three or four main courses (24–30 hours over a period of 6-8 working days) held from specialists of international renown, plus a certain number of seminars.

A C.I.M.E. Session, therefore, is neither a Symposium, nor just a School, but maybe a blend of both. The aim is that of bringing to the attention of younger researchers the origins, later developments, and perspectives of some branch of live mathematics.

The topics of the courses are generally of international resonance and the participation of the courses cover the expertise of different countries and continents. Such combination, gave an excellent opportunity to young participants to be acquainted with the most advance research in the topics of the courses and the possibility of an interchange with the world famous specialists. The full immersion atmosphere of the courses and the daily exchange among participants are a first building brick in the edifice of international collaboration in mathematical research.

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Nonlinear and Optimal Control Theory

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Preface

Mathematical Control Theory is a branch of Mathematics having as one of its main aims the establishment of a sound mathematical foundation for the control techniques employed in several different fields of applications, including engineering, economy, biology and so forth. The systems arising from these applied Sciences are modeled using different types of mathematical formalism, primarily involving Ordinary Differential Equations, or Partial Differential Equations or Functional Differential Equations. These equations depend on one or more parameters that can be varied, and thus constitute the control aspect of the problem. The parameters are to be chosen so as to obtain a desired behavior for the system. From the many different problems arising in Control Theory, the C.I.M.E. school focused on some aspects of the control and optimization of nonlinear, not necessarily smooth, dynamical systems. Two points of view were presented: Geometric Control Theory and Nonlinear Control Theory. The C.I.M.E. session was arranged in five six-hours courses delivered by Professors A.A. Agrachev (SISSA-ISAS, Trieste and Steklov Mathematical Institute, Moscow), A.S. Morse (Yale University, USA), E.D. Sontag (Rutgers University, NJ, USA), H.J. Sussmann (Rutgers University, NJ, USA) and V.I. Utkin (Ohio State University Columbus, OH, USA).

We now briefly describe the presentations.

Agrachev's contribution began with the investigation of second order information in smooth optimal control problems as a means of explaining the variational and dynamical nature of powerful concepts and results such as Jacobi fields, Morse's index formula, Levi-Civita connection, Riemannian curvature. These are primarily known only within the framework of Riemannian Geometry. The theory presented is part of a beautiful project aimed at investigating the connections between Differential Geometry, Dynamical Systems and Optimal Control Theory.

The main objective of Morse's lectures was to give an overview of a variety of methods for synthesizing and analyzing logic-based switching control systems. The term "logic-based switching controller" is used to denote a controller whose subsystems include not only familiar dynamical components

(integrators, summers, gains, etc.) but logic-driven elements as well. An important category of such control systems are those consisting of a process to be controlled, a family of fixed-gain or variable-gain candidate controllers, and an "event-drive switching logic" called a supervisor whose job is to determine in real time which controller should be applied to the process. Examples of supervisory control systems include re-configurable systems, and certain types of parameter-adaptive systems.

Sontag's contribution was devoted to the input to state stability (ISS) paradigm which provides a way of formulating questions of stability with respect to disturbances, as well as a method to conceptually unify detectability, input/output stability, minimum-phase behavior, and other systems properties. The lectures discussed the main theoretical results concerning ISS and related notions. The proofs of the results showed in particular connections to relaxations for differential inclusions, converse Lyapunov theorems, and nonsmooth analysis.

Sussmann's presentation involved the technical background material for a version of the Pontryagin Maximum Principle with state space constraints and very weak technical hypotheses. It was based primarily on an approach that used generalized differentials and packets of needle variations. In particular, a detailed account of two theories of generalized differentials, the "generalized differential quotients" (GDQs) and the "approximate generalized differential quotients" (AGDQs), was presented. Then the resulting version of the Maximum Principle was stated.

Finally, Utkin's contribution concerned the Sliding Mode Control concept that for many years has been recognized as one of the key approaches for the systematic design of robust controllers for complex nonlinear dynamic systems operating under uncertainty conditions. The design of feedback control in systems with sliding modes implies design of manifolds in the state space where control components undergo discontinuities, and control functions enforcing motions along the manifolds. The design methodology was illustrated by sliding mode control to achieve different objectives: eigenvalue placement, optimization, disturbance rejection, identification.

The C.I.M.E. course was attended by fifty five participants from several countries. Both graduate students and senior mathematicians intensively followed the lectures, seminars and discussions in a friendly and co-operative atmosphere.

As Editors of these Lectures Notes we would like to thank the persons and institutions that contributed to the success of the course. It is our pleasure to thank the Scientific Committee of C.I.M.E. for supporting our project: the Director, Prof. Pietro Zecca and the Secretary, Prof. Elvira Mascolo for their support during the organization. We would like also to thank Carla Dionisi for her valuable and efficient work in preparing the final manuscript for this volume.

Our special thanks go to the lecturers for their early preparation of the material to be distributed to the participants, for their excellent performance in teaching the courses and their stimulating scientific contributions.

We dedicate this volume to our teacher Prof. Roberto Conti, one of the pioneers of Mathematical Control Theory, who contributed in a decisive way to the development and to the international success of Fondazione C.I.M.E.

Siena and Firenze, May 2006

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