Lecture Notes in Mathematics 1931

C.I.M.E. means Centro Internazionale Matematico Estivo, that is, International Mathematical Summer Center. Conceived in the early fifties, it was born in 1954 and made welcome by the world mathematical community where it remains in good health and spirit. Many mathematicians from all over the world have been involved in a way or another in C.I.M.E.'s activities during the past years.

So they already know what the C.I.M.E. is all about. For the benefit of future potential users and co-operators the main purposes and the functioning of the Centre may be summarized as follows: every year, during the summer, Sessions (three or four as a rule) on different themes from pure and applied mathematics are offered by application to mathematicians from all countries. Each session is generally based on three or four main courses (24−30 hours over a period of 6−8 working days) held from specialists of international renown, plus a certain number of seminars.

A C.I.M.E. Session, therefore, is neither a Symposium, nor just a School, but maybe a blend of both. The aim is that of bringing to the attention of younger researchers the origins, later developments, and perspectives of some branch of live mathematics.

The topics of the courses are generally of international resonance and the participation of the courses cover the expertise of different countries and continents. Such combination, gave an excellent opportunity to young participants to be acquainted with the most advance research in the topics of the courses and the possibility of an interchange with the world famous specialists. The full immersion atmosphere of the courses and the daily exchange among participants are a first building brick in the edifice of international collaboration in mathematical research.

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Representation Theory and Complex Analysis

Lectures given at the
C.I.M.E. Summer School
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Preface

This volume collects the notes of six series of lectures given on the occasion of the CIME session *Representation Theory and Complex Analysis* held in Venice on July 10–17, 2004. We thank Venice International University for its hospitality at the beautiful venue of San Servolo island.

Our aim in organizing this meeting was to present the audience with a wide spectrum of recent results on the subject of the title, ranging from topics with an analytical flavor, to more algebraic or geometric oriented ones, without neglecting interactions with other domains, such as quantum computing.

Two papers present a general introduction to ideas and properties of analysis on semi-simple Lie groups and their unitary representations. MICHAEL COWLING presents a panorama of various interactions between representation theory and harmonic analysis on semisimple groups and symmetric spaces. Unexpected phenomena occur in this context, as for instance the Kunze–Stein property, that reveal a dramatic difference between these groups and group actions and the classical amenable group (an extension of abelian groups). Results of this type are strongly related to the vanishing of coefficients of unitary representations. Complementarily, ALAIN VALETTE recalls the notion of amenability and investigates its relations with vanishing of coefficients of unitary representations of semi-simple groups and with ergodic actions. He applies these ideas to show another surprising property of representations of semi-simple groups and their lattices, namely Margulis' super-rigidity.

Three papers deal in full detail with the hard analysis of semisimple group representations. Ideally, this analysis could be split into representations of real groups or complex groups, or of algebraic groups over local fields. A deep account of the interaction between the real and complex world is given by MASAKI KASHIWARA, whose paper studies the relations between the representation theory of real semisimple Lie groups and the (microlocal) geometry of the flag manifolds associated with the corresponding complex algebraic groups. These results, a considerable part of which are joint work with W. Schmid, were announced some years ago, and are published here in

complete form for the first time. David Vogan expresses unitary representations of real or complex semi-simple groups using tools of complex analysis, such as minimal globalizations realized on Dolbeault cohomology with compact supports. Edward Frenkel describes the geometric Langlands correspondence for complex algebraic curves, concentrating on the ramified case where a finite number of regular singular points is allowed.

Finally, Nolan Wallach illustrates briefly a surprising application that could be relevant for the future of computing and its complexity: his paper studies how representation theory is related to quantum computing, focusing attention in particular on the study of qubit entanglement.

We wish to thank all the lecturers for the excellence of their live and written contributions, as well as the many participants from all age ranges and parts of the world, who created a very pleasant working atmosphere.

Roma and Venezia, November 2006

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Contents