Lecture Notes in Mathematics 1930

José Miguel Urbano

The Method of Intrinsic Scaling

A Systematic Approach to Regularity for Degenerate and Singular PDEs

Springer

Author
José Miguel Urbano
CMUC, Department of Mathematics
University of Coimbra
3001-454 Coimbra
Portugal
jmurb@mat.uc.pt

ISBN: 978-3-540-75931-7 e-ISBN: 978-3-540-75932-4
DOI: 10.1007/978-3-540-75932-4

Lecture Notes in Mathematics ISSN print edition: 0075-8434
ISSN electronic edition: 1617-9692

Library of Congress Control Number: 2008921371

Mathematics Subject Classification (2000): 35D10, 35K65

Cover design: WMXDesign GmbH

Printed on acid-free paper

9 8 7 6 5 4 3 2 1

springer.com

To Martim, Xavier and Catarina.

Preface

When I started giving talks on regularity theory for degenerate and singular parabolic equations, a fixed-point in the conversation during the coffee-break that usually followed the seminar was the apparent contrast between the beauty of the subject and its technical difficulty. I could not agree more on the beauty part but, most of the times, overwhelmingly failed to convince my audience that the technicalities were not all that hard to follow. As in many other instances, it was the fact that the results in the literature were eventually stated and proved in their most possible generality that made the whole subject seem inexpugnable.

So when I had the chance of preparing a short course on the method of intrinsic scaling, I decided to present the theory from scratch for the simplest model case of the degenerate p-Laplace equation and to leave aside technical refinements needed to deal with more general situations. The first part of the notes you are about to read is the result of that effort: an introductory and self-contained approach to intrinsic scaling, aiming at bringing to light what is really essential in this powerful tool in the analysis of degenerate and singular equations. As another striking feature of the method is its pervasiveness in terms of the applications, in the second part of the book, intrinsic scaling is applied to several models arising from flows in porous media, chemotaxis and phase transitions. The aim is to convince the reader of the strength of the method as a systematic approach to regularity for an important and relevant class of nonlinear partial differential equations.

The analysis of degenerate and singular parabolic equations is an extremely vast and active research topic and in this contribution there is, by no means, any intention to exhaust the theory. On the contrary, the focus is on a particular subject – the (Hölder) continuity of solutions – and a unifying set of ideas. We hope that the careful study of theses notes will enable the reader to master the essential features of the method of intrinsic scaling, which is instrumental in dealing with more elaborate aspects of the theory, like the boundedness of solutions, Harnack inequalities or systems of equations.

The first four chapters contain material that would fit well in an advanced graduate course on regularity theory for partial differential equations. Each chapter corresponds roughly, with the exception of the first one, to two 90 min. classes. Chapters 5–7 are independent from one another and each could be chosen to complement the course, according to individual preferences. I would probably suggest choosing chapter 5 for that purpose.

These lecture notes had its origin in a minicourse I delivered at the 2005 Summer Program of IMPA in Rio de Janeiro. Later that year, I taught a shorter version of the course at the University of Florence. I would like to thank Marcelo Viana and Vincenzo Vespri for their kind invitations and for the wonderful hospitality. I am also indebted to all the colleagues and students who took the course for their interest and input and, in particular, to my former PhD student Eurica Henriques.

Finally, I warmly thank Emmanuele DiBenedetto for his continuing support and advice.

Coimbra, November 2007 *José Miguel Urbano*

Contents