

Lecture Notes in Mathematics

1920

Editors:

J.-M. Morel, Cachan

F. Takens, Groningen

B. Teissier, Paris

Subseries:

École d'Été de Probabilités de Saint-Flour

Saint-Flour Probability Summer School



The Saint-Flour volumes are reflections of the courses given at the Saint-Flour Probability Summer School. Founded in 1971, this school is organised every year by the Laboratoire de Mathématiques (CNRS and Université Blaise Pascal, Clermont-Ferrand, France). It is intended for PhD students, teachers and researchers who are interested in probability theory, statistics, and in their applications.

The duration of each school is 13 days (it was 17 days up to 2005), and up to 70 participants can attend it. The aim is to provide, in three high-level courses, a comprehensive study of some fields in probability theory or Statistics. The lecturers are chosen by an international scientific board. The participants themselves also have the opportunity to give short lectures about their research work.

Participants are lodged and work in the same building, a former seminary built in the 18th century in the city of Saint-Flour, at an altitude of 900 m. The pleasant surroundings facilitate scientific discussion and exchange.

The Saint-Flour Probability Summer School is supported by:

- Université Blaise Pascal
- Centre National de la Recherche Scientifique (C.N.R.S.)
- Ministère délégué à l'Enseignement supérieur et à la Recherche

For more information, see back pages of the book and
<http://math.univ-bpclermont.fr/stflour/>

Jean Picard
Summer School Chairman
Laboratoire de Mathématiques
Université Blaise Pascal
63177 Aubière Cedex
France

Steven N. Evans

Probability and Real Trees

École d'Été de Probabilités
de Saint-Flour XXXV - 2005

 Springer

Author

Steven Neil Evans
Department of Statistics # 3860
367 Evans Hall
University of California at Berkeley
Berkeley, CA 94720-3860
USA
e-mail: evans@stat.Berkeley.EDU
URL: <http://www.stat.berkeley.edu/users/evans>

Cover: Blaise Pascal (1623-1662)

Library of Congress Control Number: 2007934014

Mathematics Subject Classification (2000): 60B99, 05C05, 51F99, 60J25

ISSN print edition: 0075-8434

ISSN electronic edition: 1617-9692

ISSN Ecole d'Été de Probabilités de St. Flour, print edition: 0721-5363

ISBN 978-3-540-74797-0 Springer Berlin Heidelberg New York

DOI 10.1007/978-3-540-74798-7

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer. Violations are liable for prosecution under the German Copyright Law.

Springer is a part of Springer Science+Business Media
springer.com

© Springer-Verlag Berlin Heidelberg 2008

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Typesetting by the author and SPi using a Springer L^AT_EX macro package

Cover art: Tree of Life, with kind permission of David M. Hillis, Derrick Zwickl, and Robin Gutell, University of Texas.

Cover design: WMX Design, Heidelberg

Printed on acid-free paper SPIN: 12114894 VA41/3100/SPi 5 4 3 2 1 0

For Ailan Hywel, Ciaran Leuel and Huw Rhys

Preface

These are notes from a series of ten lectures given at the Saint-Flour Probability Summer School, July 6 – July 23, 2005.

The research that led to much of what is in the notes was supported in part by the U.S. National Science Foundation, most recently by grant DMS-0405778, and by a Miller Institute for Basic Research in Science Research Professorship.

Some parts of these notes were written during a visit to the Pacific Institute for the Mathematical Sciences in Vancouver, Canada. I thank my long-time collaborator Ed Perkins for organizing that visit and for his hospitality. Other portions appeared in a graduate course I taught in Fall 2004 at Berkeley. I thank Rui Dong for typing up that material and the students who took the course for many useful comments. Judy Evans, Richard Liang, Ron Peled, Peter Ralph, Beth Slikas, Allan Sly and David Steinsaltz kindly proof-read various parts of the manuscript.

I am very grateful to Jean Picard for all his work in organizing the Saint-Flour Summer School and to the other participants of the School, particularly Christophe Leuridan, Cedric Villani and Matthias Winkel, for their interest in my lectures and their suggestions for improving the notes.

I particularly acknowledge my wonderful collaborators over the years whose work with me appears here in some form: David Aldous, Martin Barlow, Peter Donnelly, Klaus Fleischmann, Tom Kurtz, Jim Pitman, Richard Sowers, Anita Winter, and Xiaowen Zhou. Lastly, I thank my friend and collaborator Persi Diaconis for advice on what to include in these notes.

Berkeley, California, U.S.A.

Steven N. Evans
October 2006

Contents

1	Introduction	1
2	Around the Continuum Random Tree	9
2.1	Random Trees from Random Walks	9
2.1.1	Markov Chain Tree Theorem	9
2.1.2	Generating Uniform Random Trees	13
2.2	Random Trees from Conditioned Branching Processes	15
2.3	Finite Trees and Lattice Paths	16
2.4	The Brownian Continuum Random Tree	17
2.5	Trees as Subsets of ℓ^1	18
3	\mathbb{R}-Trees and 0-Hyperbolic Spaces	21
3.1	Geodesic and Geodesically Linear Metric Spaces	21
3.2	0-Hyperbolic Spaces	23
3.3	\mathbb{R} -Trees	26
3.3.1	Definition, Examples, and Elementary Properties	26
3.3.2	\mathbb{R} -Trees are 0-Hyperbolic	32
3.3.3	Centroids in a 0-Hyperbolic Space	33
3.3.4	An Alternative Characterization of \mathbb{R} -Trees	36
3.3.5	Embedding 0-Hyperbolic Spaces in \mathbb{R} -Trees	36
3.3.6	Yet another Characterization of \mathbb{R} -Trees	38
3.4	\mathbb{R} -Trees without Leaves	39
3.4.1	Ends	39
3.4.2	The Ends Compactification	42
3.4.3	Examples of \mathbb{R} -Trees without Leaves	44
4	Hausdorff and Gromov–Hausdorff Distance	45
4.1	Hausdorff Distance	45
4.2	Gromov–Hausdorff Distance	47
4.2.1	Definition and Elementary Properties	47
4.2.2	Correspondences and ϵ -Isometries	48

4.2.3	Gromov–Hausdorff Distance for Compact Spaces	50
4.2.4	Gromov–Hausdorff Distance for Geodesic Spaces	52
4.3	Compact \mathbb{R} -Trees and the Gromov–Hausdorff Metric	53
4.3.1	Unrooted \mathbb{R} -Trees	53
4.3.2	Trees with Four Leaves	53
4.3.3	Rooted \mathbb{R} -Trees	55
4.3.4	Rooted Subtrees and Trimming	58
4.3.5	Length Measure on \mathbb{R} -Trees	59
4.4	Weighted \mathbb{R} -Trees	63
5	Root Growth with Re-Grafting	69
5.1	Background and Motivation	69
5.2	Construction of the Root Growth with Re-Grafting Process . . .	71
5.2.1	Outline of the Construction	71
5.2.2	A Deterministic Construction	72
5.2.3	Putting Randomness into the Construction	76
5.2.4	Feller Property	78
5.3	Ergodicity, Recurrence, and Uniqueness	79
5.3.1	Brownian CRT and Root Growth with Re-Grafting . . .	79
5.3.2	Coupling	82
5.3.3	Convergence to Equilibrium	83
5.3.4	Recurrence	83
5.3.5	Uniqueness of the Stationary Distribution	84
5.4	Convergence of the Markov Chain Tree Algorithm	85
6	The Wild Chain and other Bipartite Chains	87
6.1	Background	87
6.2	More Examples of State Spaces	90
6.3	Proof of Theorem 6.4	92
6.4	Bipartite Chains	95
6.5	Quotient Processes	99
6.6	Additive Functionals	100
6.7	Bipartite Chains on the Boundary	101
7	Diffusions on a \mathbb{R}-Tree without Leaves: Snakes and Spiders	105
7.1	Background	105
7.2	Construction of the Diffusion Process	106
7.3	Symmetry and the Dirichlet Form	108
7.4	Recurrence, Transience, and Regularity of Points	113
7.5	Examples	114
7.6	Triviality of the Tail σ -field	115
7.7	Martin Compactification and Excessive Functions	116
7.8	Probabilistic Interpretation of the Martin Compactification . .	122
7.9	Entrance Laws	123
7.10	Local Times and Semimartingale Decompositions	125

8	\mathbb{R}-Trees from Coalescing Particle Systems	129
8.1	Kingman's Coalescent	129
8.2	Coalescing Brownian Motions	132
9	Subtree Prune and Re-Graft	143
9.1	Background	143
9.2	The Weighted Brownian CRT	144
9.3	Campbell Measure Facts	146
9.4	A Symmetric Jump Measure	154
9.5	The Dirichlet Form	157
A	Summary of Dirichlet Form Theory	163
A.1	Non-Negative Definite Symmetric Bilinear Forms	163
A.2	Dirichlet Forms	163
A.3	Semigroups and Resolvents	166
A.4	Generators	167
A.5	Spectral Theory	167
A.6	Dirichlet Form, Generator, Semigroup, Resolvent Correspondence	168
A.7	Capacities	169
A.8	Dirichlet Forms and Hunt Processes	169
B	Some Fractal Notions	171
B.1	Hausdorff and Packing Dimensions	171
B.2	Energy and Capacity	172
B.3	Application to Trees from Coalescing Partitions	173
	References	177
	Index	185
	List of Participants	187
	List of Short Lectures	191