Part II. Regular Symplectic Reduction by Stages

In this part, we prove a general symplectic reduction by stages theorem in the case of regular (nonsingular) symplectic reduction and give several applications. Recall from the Introduction that the general setting is the following: we consider the action of a Lie group M acting freely and properly on a symplectic manifold P and form the (Meyer-Marsden-Weinstein) symplectic reduced manifold P_σ at a momentum value $\sigma \in \mathfrak{g}^*$. Assume that M has a closed normal subgroup N. A goal is to realize this reduced space P_σ in a two step procedure: first reducing by N and then by an appropriate group that is a modification of M/N. This construction generalizes and unifies several well known techniques, such as semidirect product reduction, as we shall explain in the main text. In fact, we shall give a reasonably complete discussion of the case of semidirect product reduction from the point of view of reduction by stages.

The bulk of the present Part II will make use of a special *stages hypothesis*, which is satisfied by all the examples that we are aware of. Moreover, in Chapter 11 we show that under appropriate conditions, this hypothesis is always satisfied in a large set of situations. For instance, it holds when M and N are compact groups.

Besides giving a general result in the context of symplectic manifolds, we also study the case of cotangent bundles in some detail and make use of the curvature of the mechanical connection, which provides both a magnetic term as well as the cocycle associated with the group extension.

We apply this theory to several examples, including the Heisenberg group, loop groups, the Bott–Virasoro group, and the classification of coadjoint orbits for group extensions.