Lecture Notes in Mathematics 1911

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CENTRO INTERNAZIONALE MATEMATICO ESTIVO
INTERNATIONAL MATHEMATICAL SUMMER CENTER

C.I.M.E. means Centro Internazionale Matematico Estivo, that is, International Mathematical Summer Center. Conceived in the early fifties, it was born in 1954 and made welcome by the world mathematical community where it remains in good health and spirit. Many mathematicians from all over the world have been involved in a way or another in C.I.M.E.'s activities during the past years.

So they already know what the C.I.M.E. is all about. For the benefit of future potential users and co-operators the main purposes and the functioning of the Centre may be summarized as follows: every year, during the summer, Sessions (three or four as a rule) on different themes from pure and applied mathematics are offered by application to mathematicians from all countries. Each session is generally based on three or four main courses ($24-30$ hours over a period of 6-8 working days) held from specialists of international renown, plus a certain number of seminars.

A C.I.M.E. Session, therefore, is neither a Symposium, nor just a School, but maybe a blend of both. The aim is that of bringing to the attention of younger researchers the origins, later developments, and perspectives of some branch of live mathematics.

The topics of the courses are generally of international resonance and the participation of the courses cover the expertise of different countries and continents. Such combination, gave an excellent opportunity to young participants to be acquainted with the most advance research in the topics of the courses and the possibility of an interchange with the world famous specialists. The full immersion atmosphere of the courses and the daily exchange among participants are a first building brick in the edifice of international collaboration in mathematical research.

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Alberto Bressan · Denis Serre
Mark Williams · Kevin Zumbrun

Hyperbolic Systems of Balance Laws

Lectures given at the
C.I.M.E. Summer School
held in Cetraro, Italy,
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Preface

This volume includes the lecture notes delivered at the CIME Course "Hyperbolic Systems of Balance Laws" held July 14-21, 2003 in Cetraro (Cosenza, Italy). The present volume includes lectures notes by A. Bressan, D. Serre, K. Zumbrun and M. Williams and an appendix by A. Bressan on the center manifold theorem. These are among the "hot topics" in this field and can be of great interest, not only to professional mathematicians, but also for physicists and engineers.

The concept of hyperbolic systems of balance laws was introduced by the works of natural philosophers of the eighteenth century, predominantly L. Euler (1755), and has over the past one hundred and fifty years become the natural framework for the study of gas dynamics and, more broadly, of continuum physics. During this period of time great personalities like Stokes, Challis, Riemann, Rankine, Hugoniot, Lord Rayleigh and later Prandtl, Hadamard, H. Lewy, G.I. Taylor and many others wrote several fundamental papers, thus laying the groundwork for the further development of the mathematical theory. However the first part of the past century did not see much activity on the part of mathematicians in this field and it was only during the Second World War, in connection with the Manhattan Project, that associated research received a great impetus.

Many important scientists like J. Von Neumann, R. Courant, K.O. Friedrichs, H. Bethe and Ya. Zeldowich became interested in this field and proposed many new key concepts, the influence of which remains very great to the present day.

Immediately after the Second World War there was a considerable development in mathematical theory, with key results being obtained by a new generation of great mathematicians like S.K. Godunov, P. Lax, F. John, C. Morawetz and O. Oleinik, who led the field until the mid 1960s, when J. Glimm published an outstanding paper which marked the most important breakthrough in the history of this field. Glimm was able to prove the global existence of general systems in one space dimension, with small BV data. This result introduced a new approach to nonlinear wave interaction, but the

proof was not fully deterministic. Tai-Ping Liu was later able to remove the probabilistic part of the proof, thus making it completely deterministic.

The relation between hyperbolic balance laws and continuum physics is not covered in any of the lectures in the present volume, but was the core topic of a series of lectures delivered in the Cetraro School by C. Dafermos entitled "Conservation Laws on Continuum Mechanics." In his wonderful monograph, published by Springer-Verlag in the Grundlehren der Mathematischen Wissenschaften, vol. 325, Dafermos provides an extremely thorough account of the most relevant aspects of the theory of hyperbolic conservation laws and systematically develops their ties to classical mechanics.

The notes by Alberto Bressan in this volume are intended to provide a self-contained presentation of recent results on hyperbolic conservation laws, based on the vanishing viscosity approach. Glimm's aforementioned theory was based on the construction of partially smooth approximating solutions with a locally self-similar structure. In order to get a uniform bound in BV norm, interaction potential was a crucial tool, an idea Glimm borrowed from physics. This potential, though a nonlinear functional, displays quadratic behaviour and decreases with time, provided the initial data have a small total variation.

In the 1990s Bressan and Tai-Ping Liu, together with various collaborators, completed this theory by proving the continuous dependence on initial data. The relations with the theory of compressible fluids have raised, since the very beginning of the theory, the question whether the inviscid solutions are in practice the same as the solutions with low viscosity. Although this fact had been established for various specific situations, it was only very recently that Bianchini and Bressan discovered a way to prove that, if the total variation of initial data remains sufficiently small, then the solutions of a viscous system of conservation laws converge to the solutions of the inviscid system, as long as the viscosity tends to zero. This approach allows the stability results obtained using the previous theories to be generalized.

The results are based on various technical steps, which in the present lecture notes Bressan describes in great detail, making a remarkable effort to make this difficult subject also accessible to non-specialists and young doctoral students. The notes of D. Serre cover the existence and stability of discrete shock profiles, another very exciting topic which, since the 1940s, has greatly interested applied mathematicians, including Von Neumann, Godounov and Lax, who were motivated by the need for efficient numerical codes to approximate the solutions of compressible fluid systems, including situations where shocks are present. It was immediately clear to them that a number of challenging and difficult mathematical problems needed to be solved. Partial differential equations are often approximated by finite difference schemes. The consistency and stability of a given scheme are usually studied through a linearization along elementary solutions such as constants, travelling waves, and shocks.

The study of the existence and stability of travelling waves faces significant difficulties; for example, existence may fail in rather natural situations because of small divisors problems. Ties to many branches of mathematics, ranging from dynamical systems to arithmetic number theory, prove to be relevant in this field. Serre's notes greatly emphasize this interdisciplinary aspect. His lecture notes not only provide a very useful and comprehensive introduction to this specific topic, but moreover propose a class of truly interesting and challenging problems in modern spectral theory. The analysis of the vanishing viscosity limit is far from being fully understood in the multidimensional setting. It is, in any case, important to understand the presence of stable and unstable modes along boundaries and shock profiles, where the most relevant linear and nonlinear phenomena take place. As such, the stability of viscous shock waves was the main focus of the lectures delivered by M. Williams and K. Zumbrun. This topic started, for the inviscid case, with the pioneering papers of Kreiss, Osher, Rauch and Majda. Later Metivier brought into the field a number of far-reaching ideas from microlocal analysis, in particular the paradifferential calculus introduced by Bony. The stability condition is expressed in terms of the so-called Kreiss-Lopatinskiĭ determinant. The viscous case can benefit from many of these ideas, but new tools are also needed.

Linearizing the system about a given profile (made stationary by Galilean invariance), and taking the Laplace transform in time and the Fourier transform in the hyperplane orthogonal to the direction of propagation, allows the formulation of an eigenvalue equation for a differential operator with variable coefficients. A necessary condition for the viscous profile to be stable is that these eigenvalue equations do not have (nontrivial) solutions. The Evans function technique provides a means to quantify this criterion.

But some rather subtle issues, in particular regarding regular dependence on parameters, call for a cautious approach. This was understood in a celebrated paper by J. Alexander, Gardner and C. Jones. Necessary stability conditions are expressed in terms of:
(1) the transversality of the connection in the travelling wave ODE, and
(2) the Kreiss-Lopatinskiĭ condition, which is known to ensure weak inviscid stability. The argument relies on the low frequency behaviour of the Evans function. Unlike the Kreiss-Lopatinskiĭ determinant Δ, encoding the linearized stability of the inviscid shock, the Evans function is not explicitly computable. But Zumbrun and Serre's result shows that the Evans function is tangent to Δ in the low frequency limit. Kevin Zumbrun's lectures focused on the planar stability for viscous shock waves in systems with real viscosity. His course provided an extensive overview of the technical tools and central concepts involved. He took great care to make such a difficult matter comparatively simple and approachable to an audience of young mathematicians.

M. Williams' course focused on the short time existence of curved multidimensional viscous shocks and the related small viscosity limit. It provided an accessible account of the main ideas and methods, trying to avoid most of the

technical difficulties connected with the use of paradifferential calculus. His final lecture introduced the analysis of long time stability for planar viscous shocks. A fairly complete list of references to books and research articles is included at the bottom of all of the lecture notes.

The course was attended by several young mathematicians from various European countries, who worked very hard during the whole period of the Summer School. However they were also able to enjoy the beautiful hosting facility provided by CIME, in the paradise-like sea resort of Cetraro, under the Calabrian sun.

This course was organized with the collaboration and financial support of the European Network on Hyperbolic and Kinetic Equations (HyKE).

I would like to express my gratitude to the CIME Foundation, to the CIME Director Prof. Pietro Zecca and to the CIME Board Secretary Prof. Elvira Mascolo, for their invaluable help and support, and for the tremendous efforts they have invested to return the CIME Courses to their traditional greatness.

Pierangelo Marcati

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