

Editors:

J.-M. Morel, Cachan

F. Takens, Groningen

B. Teissier, Paris

Jean-Pierre Serre

Lie Algebras and Lie Groups

1964 Lectures given at Harvard University

Corrected 5th printing

 Springer

Author

Jean-Pierre Serre
Collège de France
3, rue d'Ulm
75005 Paris, France

Mathematics Subject Classification (2000): 17B

2nd edition

Originally (1st edition) published by: W. A. Benjamin, Inc., New York, 1965
Corrected 5th printing 2006

ISSN 0075-8434

ISBN 978-3-540-55008-2 Springer-Verlag Berlin Heidelberg New York

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer. Violations are liable for prosecution under the German Copyright Law.

Springer is a part of Springer Science+Business Media
springeronline.com
© Springer-Verlag Berlin Heidelberg 1992

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Production: LE-TeX Jelonek, Schmidt & Vöckler GbR, Leipzig
Cover design: *design & production* GmbH, Heidelberg

Printed on acid-free paper SPIN: 11530756 41/3142/YL 5 4 3 2 1 0

Contents

Part I – Lie Algebras	1
Introduction	1
Chapter I. Lie Algebras: Definition and Examples	2
Chapter II. Filtered Groups and Lie Algebras	6
1. Formulae on commutators	6
2. Filtration on a group	7
3. Integral filtrations of a group	8
4. Filtrations in $GL(n)$	9
Exercises	10
Chapter III. Universal Algebra of a Lie Algebra	11
1. Definition	11
2. Functorial properties	12
3. Symmetric algebra of a module	12
4. Filtration of $U\mathfrak{g}$	13
5. Diagonal map	16
Exercises	17
Chapter IV. Free Lie Algebras	18
1. Free magmas	18
2. Free algebra on X	18
3. Free Lie algebra on X	19
4. Relation with the free associative algebra on X	20
5. P. Hall families	22
6. Free groups	24
7. The Campbell-Hausdorff formula	26
8. Explicit formula	28
Exercises	29
Chapter V. Nilpotent and Solvable Lie Algebras	31
1. Complements on \mathfrak{g} -modules	31
2. Nilpotent Lie algebras	32
3. Main theorems	33
3*. The group-theoretic analog of Engel's theorem	35
4. Solvable Lie algebras	35

5. Main theorem	36
5*. The group theoretic analog of Lie's theorem	38
6. Lemmas on endomorphisms	40
7. Cartan's criterion	42
Exercises	43
Chapter VI. Semisimple Lie Algebras	44
1. The radical	44
2. Semisimple Lie algebras	44
3. Complete reducibility	45
4. Levi's theorem	48
5. Complete reducibility continued	50
6. Connection with compact Lie groups over \mathbb{R} and \mathbb{C}	53
Exercises	54
Chapter VII. Representations of \mathfrak{sl}_n	56
1. Notations	56
2. Weights and primitive elements	57
3. Irreducible \mathfrak{g} -modules	58
4. Determination of the highest weights	59
Exercises	61
Part II – Lie Groups	63
Introduction	63
Chapter I. Complete Fields	64
Chapter II. Analytic Functions	67
“Tournants dangereux”	75
Chapter III. Analytic Manifolds	76
1. Charts and atlases	76
2. Definition of analytic manifolds	77
3. Topological properties of manifolds	77
4. Elementary examples of manifolds	78
5. Morphisms	78
6. Products and sums	79
7. Germs of analytic functions	80
8. Tangent and cotangent spaces	81
9. Inverse function theorem	83
10. Immersions, submersions, and subimmersions	83
11. Construction of manifolds: inverse images	87
12. Construction of manifolds: quotients	92
Exercises	95
Appendix 1. A non-regular Hausdorff manifold	96
Appendix 2. Structure of p -adic manifolds	97
Appendix 3. The transfinite p -adic line	101

Chapter IV. Analytic Groups	102
1. Definition of analytic groups	102
2. Elementary examples of analytic groups	103
3. Group chunks	105
4. Prolongation of subgroup chunks	106
5. Homogeneous spaces and orbits	108
6. Formal groups: definition and elementary examples	111
7. Formal groups: formulae	113
8. Formal groups over a complete valuation ring	116
9. Filtrations on standard groups	117
Exercises	120
Appendix 1. Maximal compact subgroups of $GL(n, k)$	121
Appendix 2. Some convergence lemmas	122
Appendix 3. Applications of §9: "Filtrations on standard groups" ..	124
Chapter V. Lie Theory	129
1. The Lie algebra of an analytic group chunk	129
2. Elementary examples and properties	130
3. Linear representations	131
4. The convergence of the Campbell-Hausdorff formula	136
5. Point distributions	141
6. The bialgebra associated to a formal group	143
7. The convergence of formal homomorphisms	149
8. The third theorem of Lie	152
9. Cartan's theorems	155
Exercises	157
Appendix. Existence theorem for ordinary differential equations ...	158
Bibliography	161
Problem	163
Index	165