Lecture Notes in Mathematics 1904

Peter Giesl

Construction of Global Lyapunov Functions Using Radial Basis Functions

Springer

Author

Peter Giesl
Centre for Mathematical Sciences
University of Technology München
Boltzmannstr. 3
85747 Garching bei München
Germany
e-mail: giesl@ma.tum.de

Library of Congress Control Number: 2007922353

Mathematics Subject Classification (2000): 37B25, 41A05, 41A30, 34D05

ISSN print edition: 0075-8434
ISSN electronic edition: 1617-9692
ISBN-10 3-540-69907-4 Springer Berlin Heidelberg New York
ISBN-13 978-3-540-69907-1 Springer Berlin Heidelberg New York

DOI 10.1007/978-3-540-69909-5

Springer is a part of Springer Science+Business Media
springer.com

Typesetting by the author using a Springer LaTeX macro package
Cover design: WMXDesign GmbH, Heidelberg

Printed on acid-free paper SPIN: 11979265 VA41/3100/SPi 5 4 3 2 1 0

Preface

This book combines two mathematical branches: dynamical systems and radial basis functions. It is mainly written for mathematicians with experience in at least one of these two areas. For dynamical systems we provide a method to construct a Lyapunov function and to determine the basin of attraction of an equilibrium. For radial basis functions we give an important application for the approximation of solutions of linear partial differential equations. The book includes a summary of the basic facts of dynamical systems and radial basis functions which are needed in this book. It is, however, no introduction textbook of either area; the reader is encouraged to follow the references for a deeper study of the area.

The study of differential equations is motivated from numerous applications in physics, chemistry, economics, biology, etc. We focus on autonomous differential equations $\dot{x} = f(x)$, $x \in \mathbb{R}^n$ which define a dynamical system. The simplest solutions $x(t)$ of such an equation are equilibria, i.e. solutions $x(t) = x_0$ which remain constant. An important and non-trivial task is the determination of their basin of attraction.

The determination of the basin of attraction is achieved through sublevel sets of a Lyapunov function, i.e. a function with negative orbital derivative. The orbital derivative $V'(x)$ of a function $V(x)$ is the derivative along solutions of the differential equation.

In this book we present a method to construct Lyapunov functions for an equilibrium. We start from a theorem which ensures the existence of a Lyapunov function T which satisfies the equation $T'(x) = -\bar{c}$, where $\bar{c} > 0$ is a given constant. This equation is a linear first-order partial differential equation. The main goal of this book is to approximate the solution T of this partial differential equation using radial basis functions. Then the approximation itself is a Lyapunov function, and thus can be used to determine the basin of attraction.

Since the function T is not defined at x_0, we also study a second class of Lyapunov functions V which are defined and smooth at x_0. They satisfy

the equation $V'(x) = -p(x)$, where $p(x)$ is a given function with certain properties, in particular $p(x_0) = 0$.

For the approximation we use radial basis functions, a powerful meshless approximation method. Given a grid in $\mathbb{R}^n$, the method uses an ansatz for the approximation, such that at each grid point the linear partial differential equation is satisfied. For the other points we derive an error estimate in terms of the grid density.

My Habilitation thesis [21] and the lecture “Basins of Attraction of Dynamical Systems and Algorithms for their Determination” which I held in the winter term 2003/2004 at the University of Technology München were the foundations for this book. I would like to thank J. Scheurle for his support and for many valuable comments. For their support and interest in my work I further wish to thank P. Kloeden, R. Schaback, and H. Wendland. Special thanks to A. Iske who introduced me to radial basis functions and to F. Rupp for his support for the exercise classes to my lecture. Finally, I would like to thank my wife Nicole for her understanding and encouragement during the time I wrote this book.

December 2006 *Peter Giesl*

Contents