

Wigner Matrices and Moments Estimates

In this part, we follow the strategy introduced by Wigner [205] to study the spectrum of random matrices: we estimate moments of traces of polynomials in these random matrices. We prove in this way several key results. First, we obtain the convergence (in expectation and almost surely) of the spectral measure (for the moments or the weak topology) of Wigner matrices. We also study its fluctuations around the limit. We generalize the convergence to a multi-matrix setting by showing that the trace of words in several matrices converges in the limit where the dimension goes to infinity. Finally, we generalize the estimation of moments to the case where their degree blows up with the dimension N of the matrices, but more slowly than \sqrt{N} . This is enough to bound the distance between the largest eigenvalue and its limit.