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Matrix Convolution Operators on Groups

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Preface

Recently, the non-associative algebraic analytic structures of the spaces of bounded complex harmonic functions and harmonic functionals, which are eigenfunctions of convolution operators on locally compact groups and their Fourier algebras, have been studied in detail in [13, 14]. It was proposed in [13] to further the investigation in the non-abelian matrix setting which should have wider applications. This research monograph presents some new results and developments in this connection. Indeed, we develop a general theory of matrix convolution operators on L^p spaces of matrix functions on a locally compact group G , for $1 \leq p \leq \infty$, focusing on the spectral properties of these operators and their eigenfunctions, as well as convolution semigroups, and thereby the results in [9, 13, 14] can be subsumed and viewed in perspective in this matrix context. In particular, we describe the L^p -spectrum of these operators and study the algebraic structures of eigenspaces, of which the one corresponding to the largest possible positive eigenvalue is the space of L^p matrix harmonic functions. Of particular interest are the L^∞ matrix harmonic functions which carry the structure of a Jordan triple system. We study contractivity properties of a convolution semigroup of matrix measures and its eigenspaces. Connections with harmonic functions on Riemannian manifolds are discussed.

Some results of this work have been presented in seminars and colloquia in London, Cergy-Pontoise, Hong Kong, Taiwan, Tübingen and York. We thank warmly the audience at these institutions for their inspiration and hospitality, and hope this monograph will also serve as a useful reference for the interested audience.

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