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Editors:

J.-M. Morel, Cachan

F. Takens, Groningen

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Philippe Biane · Luc Bouten · Fabio Cipriani
Norio Konno · Nicolas Privault · Quanhua Xu

Quantum Potential Theory

Editors:

Michael Schürmann
Uwe Franz

 Springer

Editors

Uwe Franz

Département de mathématiques
de Besançon

Université de Franche-Comté

16, route de Gray

25030 Besançon cedex

France

uwe.franz@univ-fcomte.fr

and

Graduate School of Information Sciences

Tohoku University

6-3-09 Aramaki-Aza-Aoba, Aoba-ku

Sendai 980-8579

Japan

Michael Schürmann

Institut für Mathematik und Informatik

Ernst-Moritz-Arndt-Universität Greifswald

Jahnstrasse 15a

17487 Greifswald

Germany

schurman@uni-greifswald.de

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Preface

This volume contains the notes of lectures given at the School “Quantum Potential Theory: Structure and Applications to Physics”. This school was held at the Alfried Krupp Wissenschaftskolleg in Greifswald from February 26 to March 9, 2007. We thank the lecturers for the hard work they accomplished in preparing and giving these lectures and in writing these notes. Their lectures give an introduction to current research in their domains, which is essentially self-contained and should be accessible to Ph.D. students. We hope that this volume will help to bring together researchers from the areas of classical and quantum probability, functional analysis and operator algebras, and theoretical and mathematical physics, and contribute in this way to developing further the subject of quantum potential theory.

We are greatly indebted to the Alfried Krupp von Bohlen und Halbach-Stiftung for the financial support, without which the school would not have been possible. We are also very thankful for the support by the University of Greifswald and the University of Franche-Comté. One of the organisers (UF) was supported by a Marie Curie Outgoing International Fellowship of the EU (Contract Q-MALL MOIF-CT-2006-022137).

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Sendai and Greifswald,
June 2008

*Uwe Franz
Michael Schürmann*

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Contributors

Philippe Biane

Institut d'électronique et d'informatique Gaspard-Monge, 77454 Marne-la-Vallée Cedex 2, France

biane@univ-mlv.fr

Luc Bouten

Caltech Physical Measurement and Control, MC 266-33, 1200 E California Blvd, Pasadena, CA 91125, USA

bouten@caltech.edu

Fabio Cipriani

Dipartimento di Matematica, Politecnico di Milano, Piazza Leonardo da Vinci, 32 I-20133 Milano, Italy

fabio.cipriani@polimi.it

Norio Konno

Department of Applied Mathematics, Yokohama National University, 79-5 Tokiwadai, Yokohama 240-8501, Japan

konno@ynu.ac.jp

Nicolas Privault

Department of Mathematics, City University of Hong Kong, 83 Tat Chee Avenue, Kowloon Tong, Hong Kong

nprivaul@cityu.edu.hk

Quanhua Xu

UFR Sciences et techniques, Université de Franche-Comté, 16, route de Gray, F-25 030 Besançon Cedex, France

quanhua.xu@univ-fcomte.fr