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Optimal Transportation Networks

Models and Theory

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Preface

The transportation problem can be formalized as the problem of finding the optimal paths to transport a measure μ^+ onto a measure μ^- with the same mass. In contrast with the Monge-Kantorovich formalization, recent approaches model the branched structure of such supply networks by an energy functional whose essential feature is to favor wide roads. Given a flow φ in a tube or a road or a wire, the transportation cost per unit length is supposed to be proportional to φ^α with $0 < \alpha < 1$. For the Monge-Kantorovich energy, $\alpha = 1$ so that it is equivalent to have two roads with flow $1/2$ or a larger one with flow 1. If instead $0 < \alpha < 1$, a road with flow $\varphi_1+\varphi_2$ is preferable to two individual roads φ_1 and φ_2 because $(\varphi_1 + \varphi_2)^\alpha < \varphi_1^\alpha + \varphi_2^\alpha$. Thus, this very simple model intuitively leads to branched transportation structures. Such a branched structure is observable in ground transportation networks, in draining and irrigation systems, in electric power supply systems and in natural objects like the blood vessels or the trees. When $\alpha > 1 - \frac{1}{N}$ such structures can irrigate a whole bounded open set of $\mathbb{R}^N$.

The aim of this set of lectures is to give a mathematical proof of several existence, structure and regularity properties empirically observed in transportation networks. This will be done in a simple mathematical framework (measures on the set of paths) unifying several different approaches and results due to Brancolini, Buttazzo, Devillanova, Maddalena, Pratelli, Santambrogio, Solimini, Stepanov, Xia and the authors.

The link with anterior discrete physical models of irrigation and erosion models in hydrography and with discrete telecommunication and transportation models will be discussed. It will be proved that most of these models fit in the simple model sketched above. Several mathematical conjectures and questions on the numerical simulation will be developed.

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Table of Contents