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Alexander Isaev

Lectures on the Automorphism Groups of Kobayashi-Hyperbolic Manifolds

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Preface

This book is based on a series of lectures that I gave at the Geometry and Analysis Seminar held in the Mathematical Sciences Institute of the Australian National University in October–November, 2005. For some time now I have been interested in characterizations of complex manifolds by their holomorphic automorphism groups, and my lectures summarized the results that I obtained in this direction (on some occasions jointly with S. Krantz and N. Kruzhilin) for the class of Kobayashi-hyperbolic manifolds during 2000–2005, with the majority of results produced in 2004–2005.

Here I give a coherent exposition (that includes complete proofs) of results describing hyperbolic manifolds for which the automorphism group dimensions are "sufficiently high" (this will be made precise in Chap. 1). The classification problem for hyperbolic manifolds with high-dimensional automorphism group can be thought of as a complex-geometric analogue of that for Riemannian manifolds with high-dimensional isometry group, which inspired many results in the 1950's-70's. Although the methods presented in the book are almost entirely different from those utilized in the Riemannian case, there is a common property that made these classifications possible: both the action of the holomorphic automorphism group of a hyperbolic manifold and the action of the isometry group of a Riemannian manifold are proper on the respective manifolds.

The book is organized as follows. In Chap. 1 I give a precise formulation of the classification problem that will be solved and summarize the main tools used in our arguments throughout the book. The classification problem splits into the homogeneous and non-homogeneous cases. The homogeneous case is treated in Chap. 2, whereas the more interesting non-homogeneous case from which the majority of manifolds arise occupies Chaps. 3–5. The general scheme for classifying non-homogeneous manifolds is, firstly, to describe all possible orbits (they turn out to have codimension 1 or 2) and, secondly, to study ways in which they can be joined together to form a hyperbolic manifold. The most challenging part of the arguments is dealing with Levi-flat and codimension 2 orbits, which becomes especially involved for 2-dimensional

hyperbolic manifolds with 3-dimensional automorphism group considered in Chap. 5.

The book concludes with a discussion of possible generalizations of the classification results to the case of not necessarily hyperbolic complex manifolds that admit proper effective actions by holomorphic transformations of Lie groups of sufficiently high dimensions (see Chap. 6).

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Canberra
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