

Editors:

A. Dold, Heidelberg

F. Takens, Groningen

Subseries: Fondazione C. I. M. E., Firenze

Advisor: Roberto Conti

Springer

Berlin

Heidelberg

New York

Barcelona

Budapest

Hong Kong

London

Milan

Paris

Santa Clara

Singapore

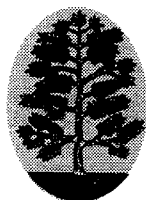
Tokyo

M. Bardi M. G. Crandall L. C. Evans
H. M. Soner P. E. Souganidis

Viscosity Solutions and Applications

Lectures given at the 2nd Session of the
Centro Internazionale Matematico Estivo
(C.I.M.E.) held in Montecatini Terme, Italy,
June 12–20, 1995

Editors: I. Capuzzo Dolcetta, P. L. Lions



Fondazione
C.I.M.E.



Springer

Authors

Martino Bardi
Dipartimento di Matematica
Università di Padova
Via Belzoni, 7
I-35131 Padova, Italy

Panagiotis E. Souganidis
Department of Mathematics
University of Wisconsin-Madison
480 Lincoln Drive
Madison, WI 53706, USA

Michael G. Crandall
Department of Mathematics
University of California
Santa Barbara, CA 93106, USA

Lawrence C. Evans
Department of Mathematics
University of California
Berkeley, CA 94720, USA

Halil Mete Soner
Department of Mathematics
Carnegie Mellon University
Schenley Park
Pittsburg, PA 15213, USA

Editors

Italo Capuzzo Dolcetta
Dipartimento di Matematica
Università di Roma "La Sapienza"
Piazzale Aldo Moro, 5
I-00185 Roma, Italy

Pierre Louis Lions
CEREMADE
Université Paris-Dauphine
Place du Maréchal de Lattre de Tassigny
F-75775 Paris Cedex 16, France

Cataloging-in-Publication Data applied for

Die Deutsche Bibliothek – CIP-Einheitsaufnahme

Viscosity solutions and applications: held in Montecatini Terme, Italy, June 12–20, 1995 / M. Bardi ... Ed.: I. Capuzzo Dolcetta; P. L. Lions. – Berlin; Heidelberg; New York; Barcelona; Budapest; Hong Kong; London; Milan; Paris; Santa Clara; Singapore; Tokyo: Springer, 1997 (Lectures given at the ... session of the Centro Internazionale Matematico Estivo (CIME) ... : 1995.2)
(Lecture notes in mathematics; Vol. 1660 : Subseries: Fondazione CIME)
ISBN 3-540-62910-6 kart.

Centro Internazionale Matematico Estivo <Firenze>: Lectures given at the ... session of the Centro Internazionale Matematico Estivo (CIME) ... – Berlin; Heidelberg; New York; London; Paris; Tokyo; Hong Kong: Springer
Früher Schriftenreihe. – Früher angezeigt u.d.T.: Centro Internazionale Matematico Estivo: Proceedings of the ... session of the Centro Internazionale Matematico Estivo (CIME)
1995.2. Viscosity solutions and applications. – 1997

Mathematics Subject Classification (1991): 35B37, 35J60, 49J15, 49K15, 49L25

ISSN 0075-8434

ISBN 3-540-62910-6 Springer-Verlag Berlin Heidelberg New York

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation, broadcasting, reproduction on microfilms or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1997
Printed in Germany

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Typesetting: Camera-ready $\text{T}_{\text{E}}\text{X}$ output by the author/editors
SPIN: 10520395 46/3142-543210 - Printed on acid-free paper

Preface

The C.I.M.E. School on Viscosity Solutions and Applications held in Montecatini from June 12 to June 20, 1995 was designed with the aim to provide a rather comprehensive and up-to-date account of the theory of viscosity solutions and some of its applications.

The School comprised the following five series of lectures:

M.G. Crandall: General Theory of Viscosity Solutions

M. Bardi: Some Applications of Viscosity Solutions to Optimal Control and Differential Games

L.C. Evans: Regularity for Fully Nonlinear Elliptic Equations and Motion by Mean Curvature

M.H. Soner: Controlled Markov Processes, Viscosity Solutions and Applications to Mathematical Finance

P.E. Souganidis: Front Propagation: Theory and Applications

as well as seminars by:

L. Ambrosio, M. Arisawa, G. Bellettini, P. Cannarsa, M. Falcone, S. Koike, G. Kossioris, M. Motta, A. Siconolfi and A. Tourin.

The present volume is a record of the material presented in the above listed courses. It is our belief that it will serve as a useful reference for researchers in the fields of fully nonlinear partial differential equations, optimal control, propagation of fronts and mathematical finance.

It is our pleasure here to thank the invited lecturers, the colleagues who contributed seminars, all the participants for their active contribution to the success of the School and the Fondazione CIME for the support in the organization.

I. Capuzzo Dolcetta, P.L. Lions

Contents

M.G. CRANDALL: Viscosity Solutions: a Primer	1
0. Introduction	1
1. On the need for nonsmooth solutions	5
2. The notion of viscosity solutions	8
3. Statement of model existence - Uniqueness theorems	9
4. Comparison for Hamilton-Jacobi equations: the Dirichlet problem	10
5. Comparison for Hamilton-Jacobi equations in \mathbb{R}^N	13
6. Hamilton-Jacobi equations in \mathbb{R}^N : unbounded solutions	15
7. Definitions revisited: semijets	17
8. Stability of the notions	19
9. Existence via Perron's method	23
10. The uniqueness machinery for second order equations	28
11. Proof of the theorem on sums	31
12. Briefly parabolic	37
 M. BARDI: Some Applications of Viscosity Solutions to Optimal Control and Differential Games	 44
Introduction	45
1. The Dynamic Programming method and Hamilton-Jacobi-Bellman equations	46
A. The classical Dynamic Programming method	
B. Viscosity solutions of Hamilton-Jacobi-Bellman equations	
2. Necessary and sufficient conditions of optimality, optimal multivalued feedbacks	58
3. Differential games	64
4. Approximations of viscosity solutions and construction of almost optimal feedbacks	71
A. Boundary conditions and weak limits in the viscosity sense	
B. Convergence of semidiscrete approximations	
5. Well-posedness of the Dirichlet problem among discontinuous solutions	81
Approach 1: Ishii's definition and the "complete solution"	
Approach 2: Bilateral supersolutions	
Approach 3: Envelope solutions	

L.C. EVANS: Regularity for Fully Nonlinear Elliptic Equations and Motion by Mean Curvature	98
Introduction	98
1. Regularity for fully nonlinear Pde of second order	99
A. Introduction,viscosity solutions,linear estimates	
B. $C^{1,\alpha}$ estimates	
C. $C^{2,\alpha}$ estimates	
2. Motion by mean curvature	115
D. Introduction,distance function	
E. Level set method	
F. Geometric properties	
 M.H. SONER: Controlled Markov Processes, Viscosity Solutions and Applications to Mathematical Finance	134
1. Controlled diffusion processes	134
1.1 Stochastic differential equations	
1.2 Controlled diffusion processes	
1.3 Dynamic Programming:formal description	
1.4 Verification theorems	
1.5 Dynamic Programming principle	
1.6 Viscosity solutions	
1.7 Boundary conditions	
2. Singular Stochastic Control	153
2.1 Formal discussion	
2.2 Reformulation	
2.3 Verification theorem	
2.4 Viscosity solutions	
3. Portfolio Selection with Transaction Costs	164
3.1 Problem	
3.2 Regularity	
3.3 Optimal investment-transaction policy	
3.4 Other related problems	
4. Pricing a European Option	172
4.1 Black - Scholes formula	
4.2 Option pricing via utility maximization	
4.3 Transaction costs	
4.4 Leland's approach to transaction costs	

P.E. SOUGANIDIS: Front Propagation:	
Theory and Applications	186
0. Introduction	186
1. Generalized Front Propagation	188
2. An Abstract Method	204
3. Asymptotics of bistable type - Phase theory	208
3.1 Rigorous results	
3.2 Some new results	
4. Asymptotics of reaction diffusion equations and systems of KPP-type	220
4.1 General theory	
4.2 Some results about systems	
4.3 Front dynamics for turbulent reaction-diffusion equations	
5. Macroscopic (hydrodynamic) limits of stochastic Ising models with long range interactions	231
5.1 Equilibrium theory	
5.2 Dynamic (nonequilibrium) theory mesoscopic limit	
5.3 Macroscopic behavior of the mean field equation	
5.4 Macroscopic (hydrodynamic) limit for the stochastic Ising model	