

# Lecture Notes in Mathematics

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## Saint-Flour Probability Summer School



The Saint-Flour volumes are reflections of the courses given at the Saint-Flour Probability Summer School. Founded in 1971, this school is organised every year by the Laboratoire de Mathématiques (CNRS and Université Blaise Pascal, Clermont-Ferrand, France). It is intended for PhD students, teachers and researchers who are interested in probability theory, statistics, and in their applications.

The duration of each school is 13 days (it was 17 days up to 2005), and up to 70 participants can attend it. The aim is to provide, in three high-level courses, a comprehensive study of some fields in probability theory or Statistics. The lecturers are chosen by an international scientific board. The participants themselves also have the opportunity to give short lectures about their research work.

Participants are lodged and work in the same building, a former seminary built in the 18th century in the city of Saint-Flour, at an altitude of 900 m. The pleasant surroundings facilitate scientific discussion and exchange.

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# Stability of Queueing Networks

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## Preface

These notes include the material from a series of nine lectures given at the Saint-Flour Probability Summer School, July 2 - July 15, 2006. Lectures were also given by Alice Guionnet and Steffen Lauritzen. It was my first visit to Saint-Flour, which I enjoyed considerably.

The topic of these notes, the stability of queueing networks, has been of interest to the queueing community since the early 1990s. At that point, little was known about this and other aspects of queueing networks in general settings. There is now a theory (albeit incomplete), with positive criteria for stability as well as examples where such stability fails. I felt that the time was right to combine in one place a summary of such work.

I wish to thank Jean Picard for his work in organizing the Saint-Flour Summer School and the other participants of the School. I thank the following individuals with whom I consulted at various points: John Baxter, Jim Dai, Michael Harrison, John Hasenbein, Haya Kaspi, Thomas Kurtz, Sean Meyn, and Ruth Williams. I also thank John Baxter for his technical help in preparing the manuscript.

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Minneapolis, Minnesota, U.S.A.

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March 2008

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