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Probabilistic Models for Nonlinear Partial Differential Equations

Lectures given at the 1st Session of the
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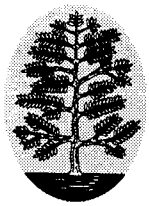

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Preface

These last years, there have been important developments in the probabilistic interpretation of nonlinear Partial Differential Equations, the theory of the convergence of law of stochastic processes and the numerical approximation of stochastic processes.

All these developments offer the appropriate theoretical background to analyse probabilistic algorithms used to solve equations as important in practice as the Navier-Stokes equation, the Boltzmann equation and certain Stochastic Partial Differential Equations. They also permit us to construct new methods.

For example, all the works around the propagation of chaos, particularly those of A-S. Sznitman, permit a quite new and fruitful point of view on the random vortex methods in Fluid Mechanics and on Monte–Carlo methods for Boltzmann-like equations. Likewise, the numerical analysis of stochastic differential equations has recently progressed in several interesting directions (variance reduction techniques, simulation of reflected diffusion processes, convergence in law of the normalized trajectorial error, asymptotic expansions of the discretization error).

Weak limit theorems for stochastic integrals naturally are among the main ingredients in the study of interacting particle systems, approximation procedures for solutions of stochastic differential equations, etc. A selection of such theorems in view of the analysis of applied problems should be useful. Besides, quite new weak limit theorems have just appeared for stochastic integrals with respect to infinite dimensional semimartingales.

We therefore enthusiastically answered Prof. V. Capasso's suggestion to submit to the CIME a proposal for a Summer School on the probabilistic models for nonlinear PDE's and their numerical applications with a three-fold emphasis: first, on the weak convergence of stochastic integrals; second, on the probabilistic interpretation and the particle approximation of equations coming from Physics (conservation laws, Boltzmann-like and Navier-Stokes equations); third, on the modelling of networks by interacting particle systems.

We thank all the participants to this Summer School which was held in Montecatini from May 22^{th} to May 30^{th}. The exchanges between the lecturers and the audience were very useful for everybody.

We thank all the lecturers (Carl Graham, Tom Kurtz, Sylvie Méléard, Philip Protter, Mario Pulvirenti) for having given fascinating lectures and for having written pedagogic and deep contributions to the present volume.

We hope that this book will be useful for our colleagues working on stochastic particle methods and on the approximation of SPDE's and in particular, for Ph.D. students and for young researchers.

We thank CIME for its generous financial support and for arranging the location in Montecatini offering us the combined delights of beautiful Tuscan surroundings and gastronomical excellence.

Sophia–Antipolis and Trento, December 1995

Denis Talay and Luciano Tubaro

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