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B. Cockburn C. Johnson
C.-W. Shu E. Tadmor

Advanced Numerical Approximation of Nonlinear Hyperbolic Equations

Lectures given at the 2nd Session of the
Centro Internazionale Matematico Estivo
(C.I.M.E.) held in Cetraro, Italy,
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Authors

Bernardo Cockburn
School of Mathematics
University of Minnesota
Minneapolis, MN 55455, USA
e-mail: cockburn@math.umn.edu

Claes Johnson
Mathematics Department
Chalmers University of Technology
S-41296 Göteborg, Sweden
e-mail: claes@math.chalmers.se

Chi-Wang Shu
Division of Applied Mathematics
Brown University
Providence, RI 02912, USA
e-mail: shu@cfm.brown.edu

Eitan Tadmor
Department of Mathematics UCLA
Los Angeles, CA 90095, USA
and
School of Mathematical Sciences
Tel-Aviv University
Tel-Aviv, Israel
e-mail: tadmor@math.ucla.edu

Editor

Alfio Quarteroni
Dipartimento di Matematica
Politecnico di Milano
Via Bonardi 9
I-20133 Milano, Italy
e-mail: aq@mate.polimi.it

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Preface

The C.I.M.E. School on *Advanced Numerical Approximation on Nonlinear Hyperbolic Equations*, held in Cetraro (Italy) from June 23rd to June 28th, 1997, aimed to provide a comprehensive and up-to-date presentation of numerical methods which are used nowadays to solve nonlinear partial differential equations of hyperbolic type, developing shock discontinuities. The lectures were given by four outstanding scientists in the field and reflect the state of the art of a broad spectrum of topics. The most modern and effective methodologies in the framework of finite elements, finite differences, finite volumes, spectral methods and kinetic methods are addressed. In particular, the following approaches are considered: high-order shock capturing techniques, discontinuous space-time finite elements, discontinuous Galerkin methods, adaptive techniques based upon a-posteriori error analysis. The theoretical properties of each method and its algorithmic complexity are addressed. A wide variety of applications to the solution of systems of conservation laws arising from fluid dynamics and other fields is considered.

This volume collects the texts of the four series of lectures presented at the Summer School. While they are ordered alphabetically by author, the book opens with the lecture of Prof. Eitan Tadmor, as it contains an introductory overview to the subject which can serve as an introduction for the other lectures in this volume.

It is my pleasure as editor of these Lecture Notes to thank the Director and the Members of the C.I.M.E. Scientific Committee, in particular Prof. Arrigo Cellina for the invitation to organize the School and their support during the organization and to the C.I.M.E. staff, lead by Prof. Pietro Zecca. My very sincere thanks go to the lecturers for their excellent job of preparing and teaching the Course and a preliminary version of the lectures to be distributed among the participants. Particular thanks go to all the participants for having created an extraordinarily friendly and stimulating atmosphere, and to those who have contributed with short communications: Tim Barth, Angelo Iollo, Stefano Micheletti, Gabriella Puppo, Giovanni Russo, Riccardo Sacco, Fausto Saleri and Alessandro Veneziani. Finally, I would like to thank the Director and staff of the “Grand Hotel San Michele” in Cetraro (Cosenza) for the kind hospitality and efficiency and the following collaborators for their invaluable help: Simona Lilliu from CRS4 (our scientific secretary), Francesco Bosio, Simona Perotto and Alessandro Veneziani from the Politecnico di Milano, for their careful editing of the manuscripts.

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