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# The Cauchy Problem for Higher-Order Abstract Differential Equations

Springer

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To Our Motherland

To our parents and teachers

To Xiao Liang

# Preface

The main purpose of this book is to present the basic theory and some recent developments concerning the Cauchy problem for higher order abstract differential equations

$$\begin{cases} u^{(n)}(t) + \sum_{i=0}^{n-1} A_i u^{(i)}(t) = 0, \quad t \geq 0, \\ u^{(k)}(0) = u_k, \quad 0 \leq k \leq n-1. \end{cases} \qquad (ACP_n)$$

where $A_0, A_1, \cdots, A_{n-1}$ are linear operators in a topological vector space $E$.

Many problems in nature can be modeled as $(ACP_n)$. For example, many initial value or initial-boundary value problems for partial differential equations, stemmed from mechanics, physics, engineering, control theory, etc., can be translated into this form by regarding the partial differential operators in the space variables as operators $A_i$ $(0 \leq i \leq n-1)$ in some function space $E$ and letting the boundary conditions (if any) be absorbed into the definition of the space $E$ or of the domain of $A_i$ (this idea of treating initial value or initial-boundary value problems was discovered independently by E. Hille and K. Yosida in the forties). The theory of $(ACP_n)$ is closely connected with many other branches of mathematics. Therefore, the study of $(ACP_n)$ is important for both theoretical investigations and practical applications.

Over the past half a century, $(ACP_n)$ has been studied extensively. Especially for $(ACP_1)$, the theory (or closely related operator semigroup theory) has evolved comparatively perfect since the well-known Hille-Yosida theorem came out in 1948, and is well documented in the monographs of, e.g., Brézis [1], Davies [1], deLaubenfels [9], Fattorini [6], Goldstein [7], Hille-Phillips [1], Nagel [2], Pazy [2], van Casteren [1], van Neerven [1], Yosida [4]. On the other hand, since the work of Lions [1] in 1957, the study of higher order $(ACP_n)$ $(n \geq 2)$ has also received much attention (cf., e.g., Fattorini [7], Goldstein [7], Krein [1], Xiao [2] and references therein). So far, a rich theory of $(ACP_n)$, including the Hille-Yosida type characterization for wellposed $(ACP_n)$ of higher order, has unfolded before us.

A survey of the research history of $(ACP_n)$ $(n \geq 2)$ shows that one popular approach is to reduce the higher order problem to a first order system in a suitable phase space and use operator semigroup theory. The disadvantage of this approach is that, finding an ideal phase space is generally difficult, and the structure of the phase space (if any) may be complicated so that inconvenient

to application; also some inherent properties of higher order problems can not always be reflected precisely from the corresponding first order systems. The strong desire to establish concise, convenient and more inclusive theories about $(ACP_n)$ therefore gives rise to another approach — direct treatment of $(ACP_n)$ $(n \geq 2)$. This book depends heavily on this second idea, with the aid of the first one when needed.

The main material in this book is taken from the authors' work on this topic. We have tried to give a systematic exposition of the abstract theory of $(ACP_n)$, but no attempt at completeness can be made at this time, either in the text or in the references. Actually, many results and papers have not been mentioned. Also we do not attempt to give detailed applications, although many results are illustrated with concrete examples.

As prerequisites for the reading of this book we assume the reader to have a sound knowledge of complex and functional analysis. Familiarity with the basic theory of operator semigroups is desirable but not necessary. Some basic facts for the fractional powers of closed operators, and for the classical strongly continuous operator semigroups as well as cosine operator functions, which are needed in this book, are gathered in Appendix. For other preliminaries, we have dispensed with a special chapter of them in favour of reminders in the body of the text and where necessary we refer to other books and papers for background material.

Chapters 1 and 2 are presented mainly in the setting of sequentially complete locally convex spaces, while other chapters in Banach spaces. Throughout, the method of Laplace transforms will be a fundamental tool. So we firstly in Chapter 1 discuss basic properties of Laplace transforms, especially the integrated version of the Widder's classical representation theorem for Laplace transforms (Theorem 2.1). In addition, we give a brief introduction of the basics of integrated and regularized semigroups or cosine functions, as well as their relationship to abstract Cauchy problems. Chapter 2 is devoted to the establishment of the Hille-Yosida type characterization of strongly wellposed $(ACP_n)$, and others. Chapter 3 selects to deal with several types of $(ACP_n)$ which are not wellposed in a standard sense. Chapters 4 – 7 investigate various properties of the propagators or solutions of $(ACP_n)$, including analyticity, parabolicity, exponential growth bound, exponential stability, differentiability, norm continuity, and almost periodicity; corresponding characterizations are given.

Within each chapter definitions, lemmas, theorems, corollaries, etc. are numbered consecutively as 1.1, 1.2, $\cdots$, in section x.1 (x=1, 2, $\cdots$, 7), as 2.1, 2.2, $\cdots$, in section x.2 (x=1, 2, $\cdots$, 7) and so on. When making a reference to another chapter we always add the number of that chapter, e.g., 2.1.2.

Throughout this book, $N$, $N_0$, $R$, $R^+$, $\mathbf{C}$ denote the positive integers, the nonnegative integers, the real numbers, the nonnegative real numbers, the complex plane, respectively. For $b \in R$, $[b]$ will be the least integer$> b-1$. Let $E$ and $X$ be topological vector spaces. $\mathrm{L}(E,\ X)$ denotes the space of all continuous linear operators from $E$ to $X$, and $\mathrm{L}(E,\ E)$ will be abbreviated to $\mathrm{L}(E)$. If $E$ is a locally convex space topologized by the family $\Gamma$ of seminorms, we denote by $\mathcal{B}_\Gamma(E)$ the

subspace of $\mathrm{L}(E)$ whose elements $B$ satisfy $\|B\|_\Gamma := \sup\{p(Bx);\ p \in \Gamma,\ x \in E$ with $p(x) \leq 1\} < \infty$; $\mathcal{B}_\Gamma(E)$ with norm $\|\cdot\|_\Gamma$ is a normed algebra. For $k \in N_0$, $C^k(R^+,\ E)$ is the set of all $k$-times continuously differentiable $E$-valued functions in $R^+$; $C(R^+,\ E) := C^0(R^+,\ E)$; $C^\infty(R^+, E) := \bigcap_{k=0}^\infty C^k(R^+, E)$. For a linear operator $A$, we will write $\mathcal{D}(A)$, $\mathcal{R}(A)$, $\mathcal{N}(A)$, $\sigma(A)$, $\sigma_p(A)$, $\rho(A)$, $R(\lambda;\ A)$, $A^*$, respectively, for the domain, the image, the kernel, the spectrum, the point spectrum, the resolvent set, the resolvent, the adjoint operator. Finally, the characteristic polynomial of the equation in $(ACP_n)$ is denoted by

$$P_\lambda := \lambda^n + \sum_{i=0}^{n-1} \lambda^i A_i, \qquad \lambda \in \mathbf{C},$$

and

$$R_\lambda := P_\lambda^{-1},$$

if the inverse exists. For each $0 \leq k \leq n-1$, $\underset{\sim}{A}_k$ will denote the restriction of $A_k$ on $\bigcap_{i=0}^k \mathcal{D}(A_i)$. Sometimes we write $A_n := I$ (the identity operator).

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Jin Liang

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