

Lecture Notes in Mathematics

1609

Editors:

A. Dold, Heidelberg

F. Takens, Groningen

Subseries: Fondazione C.I.M.E., Firenze

Advisor: Roberto Conti

Springer

Berlin

Heidelberg

New York

Barcelona

Budapest

Hong Kong

London

Milan

Paris

Tokyo

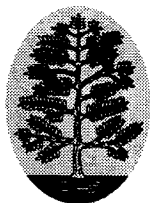
L. Arnold C. Jones
K. Mischaikow G. Raugel

Dynamical Systems

Lectures given at the 2nd Session of the
Centro Internazionale Matematico Estivo
(C.I.M.E.)

held in Montecatini Terme, Italy,
June 13-22, 1994

Editor: R. Johnson



Fondazione
C.I.M.E.



Springer

Authors

Ludwig Arnold
Institut für Dynamische Systeme
Universität Bremen
D-28334 Bremen, Germany

Christopher K. R. T. Jones
Division of Applied Mathematics
Brown University
Providence, RI 02912, USA

Konstantin Mischaikow
Department of Mathematics
Georgia Institute of Technology
Atlanta, GA 30332, USA

Geneviève Raugel
Laboratoire d'Analyse Numérique
Université de Paris-Sud
F-91405 Orsay, France

Editor

Russell Johnson
Dipartimento di Sistemi e Informatica
Università di Firenze
Via di S. Marta 3
I-50139 Firenze, Italy

Mathematics Subject Classification (1991): 34C35, 34C37, 34E15, 35B40, 35K55, 36Q30, 54H20, 58F12, 60H10, 60H25

ISBN 3-540-60047-7 Springer-Verlag Berlin Heidelberg New York

Dynamical systems : held in Montecatini Terme, Italy, June 13-22, 1994 / L. Arnold ... Ed.: R. Johnson. – Berlin ; Heidelberg ; New York ; Paris ; Tokyo ; Hong Kong : Springer, 1995
(Lectures given at the ... session of the Centro Internazionale Matematico Estivo (CIME) ... ; 1994,2) (Lecture notes in mathematics ; Vol. 1609 : Subseries: Fondazione CIME)
ISBN 3-540-60047-7 (Berlin ...)
ISBN 0-387-60047-7 (New York ...)
NE: Arnold, Ludwig; Johnson, Russell [Hrsg.]; 2. GT

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation, broadcasting, reproduction on microfilms or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1995
Printed in Germany

Typesetting: Camera-ready by authors
SPIN: 10130360 46/3142-543210 - Printed on acid-free paper

Preface

This volume contains the lecture notes written by the four principal speakers at the C.I.M.E. session on Dynamical Systems held at Montecatini in June 1994. A goal of the session was to illustrate how methods of dynamical systems can be applied to the study of differential equations. And indeed the reader will find discussed in these notes a broad range of techniques recently developed in dynamical systems theory together with numerous applications to problems, both conceptual and specific, in ordinary and partial differential equations.

The lectures were delivered in the format of mini-courses of six hours each by Prof. Geneviève Raugel of the Université de Paris-Sud, Prof. Konstantin Mischaikow of Georgia Tech University, Prof. Christopher Jones of Brown University, and Prof. Ludwig Arnold of the Universität Bremen. They were attended by scholars from Italy and several other countries; their good-natured and active participation was essential to the success of the session.

As the reader will discover, the content of each of the courses is very rich. The remarks which follow are for purposes of general orientation only. For an adequate outline of the material in the courses the reader is referred to the statements written by the speakers themselves.

Prof. Arnold outlined a theory of random dynamical systems, discussing foundational points and applications to ordinary differential equations with random coefficients. He gave results on invariant manifolds and normal forms, and among other applications considered the breakdown of stability of a fixed point in certain differential equations with stochastic coefficients.

Prof. Jones reviewed the approach to singular perturbation theory recently developed by him and his collaborators. This approach is based on certain fundamental structure results of Fenichel together with the "exchange lemma". These matters and other were thoroughly discussed and numerous applications were given.

Prof. Mischaikow defined the Conley index of an isolated invariant set in a flow, then went on to consider the nature of the corresponding Morse sets and the set of connecting orbits. He reviewed a recent application of the Conley index theory: a proof of the existence of chaos in the Lorenz equations for certain parameter values.

Prof. Raugel considered recent work in the theory of partial differential equations and their singular perturbations. In particular she discussed partial differential equations in thin domains and the semi-continuity properties of attractors for such equations. She illustrated the theory by applications to the damped wave equation, the Navier-Stokes equations, and others.

Thanks are due to the main lecturers for the care they took in preparing their courses and for the speed with which they readied their notes for publication.

Thanks are also due to the six other participants of the session who delivered single lectures. Unfortunately it was not possible to publish their contributions in this volume. However a list of the titles of their lectures is given in the succeeding pages.

Finally, all participants, and especially the organizer, are indebted to the Director of the C.I.M.E., Prof. Roberto Conti, the Secretary Prof. Pietro Zecca, and to the C.I.M.E. staff for cooperation manifested in many ways.

Russell Johnson

TABLE OF CONTENTS

L. ARNOLD	Random Dynamical Systems	1
C.K.R.T. JONES	Geometric Singular Perturbation Theory	44
K. MISCHAIKOW	Conley Index Theory	119
G RAUGEL	Dynamics of Partial Differential Equations on Thin Domains	208