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# Fluctuation Theory for Lévy Processes

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## Foreword

The Saint-Flour Probability Summer School was founded in 1971. It is supported by CNRS, the “Ministère de la Recherche”, and the “Université Blaise Pascal”.

Three series of lectures were given at the 35th School (July 6–23, 2005) by the Professors Doney, Evans and Villani. These courses will be published separately, and this volume contains the course of Professor Doney. We cordially thank the author for the stimulating lectures he gave at the school, and for the redaction of these notes.

53 participants have attended this school. 36 of them have given a short lecture. The lists of participants and of short lectures are enclosed at the end of the volume.

Here are the references of Springer volumes which have been published prior to this one. All numbers refer to the *Lecture Notes in Mathematics* series, except S-50 which refers to volume 50 of the *Lecture Notes in Statistics* series.

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Further details can be found on the summer school web site  
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Jean Picard  
Clermont-Ferrand, April 2006

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