

Lecture Notes in Mathematics

1567

Editors:

A. Dold, Heidelberg

B. Eckmann, Zürich

F. Takens, Groningen

Subseries:

Nankai Institute of Mathematics,
Tianjin, P. R. China (vol. 10)

Advisor:

S. S. Chern, B.-j. Jiang



R.L. Dobrushin S. Kusuoka

Statistical Mechanics and Fractals

Springer-Verlag

Berlin Heidelberg New York

London Paris Tokyo

Hong Kong Barcelona

Budapest

Authors

Roland Lvovich Dobrushin
Institute for Problems of Information Transmission
Ermolovoj 19
103051 Moscow, Russia

Shigeo Kusuoka
Research Institute of Mathematical Sciences
Kyoto University
606 Kyoto, Japan

New address:
Department of Mathematical Sciences
University of Tokyo
113 Tokyo, Japan

Mathematics Subject Classification (1991): 82BXX, 82B31, 60K35, 60J60

ISBN 3-540-57516-2 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-57516-2 Springer-Verlag New York Berlin Heidelberg

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation, broadcasting, reproduction on microfilms or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1993
Printed in Germany

2146/3140-543210 - Printed on acid-free paper

FOREWORD

The Nankai Institute of Mathematics held a special Year in Probability and Statistics during the academic year 1988-1989. We had over 150 specialists, professors and graduate students, who participated in this Special Year from August 1988 to May 1989. More than twenty outstanding probabilists and statisticians from several countries were invited to give lectures and talks. This volume contains two lectures, one is written by Professor R. L. Dobrushin, and the other one by Professor S. Kusuoka.

We would like to express our gratitude to Professors Dobrushin and Kusuoka for their enthusiasm and cooperation.

Ze-Pei Jiang

Shi-Jian Yan

Ping Cheng

Rong Wu

TABLE OF CONTENTS

Part 1. On the Way to the Mathematical Foundations of Statistical Mechanics

by R. L. Dobrushin

§0. Introduction	1
§1. Realization of the Classical Fluid Model	5
§2. Dynamics of a Finite System	7
§3. Dynamics of an Infinite System	14
§4. Random Evolution	19
§5. Gibbsian States in Finite Volumes	21
§6. Gibbsian Measures in an Infinite Volume	26
§7. Random Evolution (revisited)	28
§8. Hydrodynamical Equations	31
References	36

Part 2. Diffusion Processes on Nested Fractals by S. Kusuoka

Introduction	39
§1. Self-similar Fractal	41
§2. Nested fractals and their geometrical properties	47
§3. Transition probability of Markov chain	54
§4. Dirichlet form on nested fractal	62
§5. Probability measure induced by random matrices	72
§6. Expression of Dirichlet form	78
§7. Some remarks for the measure μ	87
References	97