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Dynamical Systems and Small Divisors

Lectures given at the C.I.M.E. Summer School held in Cetraro, Italy, June 13 - 20, 1998

Editors: S. Marmi
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Preface

The C.I.M.E. Session "Dynamical Systems and Small Divisors" was held in Cetraro (Cosenza, Italy) from June 13 to June 20, 1998.

Lecture series were held by:

- L.H. Eliasson (8 hours): Linear quasiperiodic systems
- S. Kuksin (3 hours): KAM theory and PDEs
- M.R. Herman (8 hours): Abstract methods in small divisors: Implicit function theorems in Fréchet spaces
- J.N. Mather (3 hours): Variational construction of orbits
- J.-C. Yoccoz (8 hours): Geometrical methods in small divisor problems

In addition there was a 3 hours long open problem session.
Furthermore, the following participants gave a talk:

1. U. Bessi: A counterexample to KAM theorem
2. D. De Latte: Linearization of commuting holomorphic maps
3. G. Gentile: Scaling properties near resonances for the (semistandard and) standard map
4. H. Ito: Integrable symplectic maps and their Birkhoff normal forms
5. R. Krikorian: Reducibility of linear quasiperiodic systems: a global density result
6. P. Moussa: Regularity properties of Brjuno functions
7. D. Sauzin: Quasianalytic monogenic solutions of a cohomological equation
8. S. Smirnov: Weak expansion and geometry of Julia sets
9. L. Stolovitch: Singular complete integrability
10. M. Vittot: Perturbation method for Floquet hamiltonians with dense point spectrum

This volume contains expanded versions of the lecture series of Eliasson, Kuksin and Yoccoz and of the open problem session.

Many problems of stability in the theory of dynamical systems face the difficulty of small divisors. The most famous example is probably given by Kolmogorov–Arnol'd–Moser theory on the persistence of quasi-periodic solutions of Hamilton's equations for quasi-integrable Hamiltonian systems (both finite and infinite-dimensional, like nonlinear wave equations). This is a very natural situation with many applications to physics and astronomy. What all these different problems have in common is roughly speaking what follows: one can associate some "frequencies" to the orbits under investigation and some arithmetical condition is needed to prove their existence and stability. In

the smooth case one must impose a diophantine condition on the frequencies, in the analytic (finite–dimensional) case one can impose a weaker condition.

For one-dimensional analytic small divisor problems the optimal arithmetical condition to impose is now known: in the local case it is the same Brjuno condition introduced some 30 years ago, whereas in the global case one has to impose a more restrictive condition: the lectures by J.-C. Yoccoz published in these notes deal with these cases.

In his lectures Hakan Eliasson addresses the problem of reducibility of linear quasiperiodic skew-products. Originating from the pioneering works of Dinaburg–Sinai and Rüssmann, who constructed Floquet solutions for the continuous Schrödinger equation with a quasiperiodic potential in the so-called weak coupling regime (the potential is "small" w.r.t. the kinetic part), many authors have since then studied more general situations (strong coupling, discrete case, more general groups, etc.). The review of Eliasson at the ICM 1998 could serve as a general introduction to this topic.

The contribution of S. Kuksin is devoted to a proof of KAM theorem of persistence of finite–dimensional tori under small Hamiltonian perturbations of Lax-integrable Hamiltonian PDEs. Here the model problem is provided by Korteweg–de Vries equation $u_t = -u_{xxx} + 6uu_x$ where the spatial variable x varies on the one–dimensional torus. Also other problems can be treated by the same method: sine-Gordon equation, (φ^4)-equation, etc.. The study of nonlinear Hamiltonian PDEs is a rapidly growing research subject: excellent reviews of recent work are provided by the addresses of Bourgain at the ICM 1994 and Kuksin at the ICM 1998.

Finally, the open problems proposed by the speakers have been collected in the last contribution. They deal both with finite and infinite–dimensional small divisor problems. As far as possible we have tried to make this text self-contained.

We are grateful to all speakers and partecipants for their essential contribution to the success of this C.I.M.E. session. Special thanks go to Roberta Fabbri who helped us in the organization and to Raphael Krikorian who held several tutorials on the material of Herman's lectures.

Stefano Marmi and Jean-Christophe Yoccoz

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