Lecture Notes in Mathematics 830

J.A. Green

Polynomial Representations of GL_n

2nd corrected and augmented edition

with an Appendix on Schensted Correspondence and Littelmann Paths

by K. Erdmann, J.A. Green and M. Schocker

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Preface to the second edition

This second edition of "Polynomial representations of $\mathsf{GL}_n(K)$" consists of two parts. The first part is a corrected version of the original text, formatted in LaTeX, and retaining the original numbering of sections, equations, etc. The second is an Appendix, which is largely independent of the first part, but which leads to an algebra $L(n,r)$, defined by P. Littelmann, which is analogous to the Schur algebra $S(n,r)$. It is hoped that, in the future, there will be a structure theory of $L(n,r)$ rather like that which underlies the construction of Kac-Moody Lie algebras.

We use two operators which act on "words". The first of these is due to C. Schensted (1961). The second is due to Littelmann, and goes back to a 1938 paper by G. de B. Robinson on the representations of a finite symmetric group. Littelmann's operators form the basis of his elegant and powerful "path model" of the representation theory of classical groups. In our Appendix we use Littelmann's theory only in its simplest case, i.e. for GL_n.

Essential to my plan was to establish two basic facts connecting the operations of Schensted and Littelmann. To these "facts", or rather conjectures, I gave the names Theorem A and Proposition B. Many examples suggested that these conjectures are true, and not particularly deep. But I could not prove either of them.

This work was therefore stalled, until I sought the help of my colleagues Karin Erdmann and Manfred Schocker. They accepted the challenge, and within a few weeks produced proofs of both conjectures. Their proofs constitute the heart of the Appendix, and make it possible to begin a comparison of the Littelmann algebra $L(n,r)$ with the Schur algebra $S(n,r)$. Karin and Manfred have made this Appendix possible, and have written large parts of the text. It has been a happy experience for me to work with them.

A few weeks before the final manuscript of the Appendix was ready, we heard that A. Lascoux, B. Leclerc and J.-Y. Thibon have published a work

on “The plactic monoid”, which contains results equivalent to Theorem A and Proposition B. Their methods are rather different from ours, and they prove also many important facts which do not come into our Appendix. We give a brief summary of this work in §D.11.

Oxford, August 2006 Sandy (J. A.) Green

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