Lecture Notes in Mathematics 1407

Wolfram Pohlers

Proof Theory

An Introduction

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Preface

This book contains the somewhat extended lecture notes of an introductory course in proof theory I gave during the winter term 1987/88 at the University of Münster, FRG. The decision to publish these notes in the Springer series has grown out of the demand for an introductory text on proof theory. The books by K.Schütte and G.Takeuti are commonly considered to be quite advanced and J.Y.Girard's brilliant book also, is too broad to serve as an introduction.

I tried, therefore, to write a book which needs no previous knowledge of proof theory at all and only little knowledge in logic. This is of course impossible, so the book runs on two levels - a very basic one, at which the book is self-contained, and a more advanced one (chiefly in the exercises) with some cross-references to definability theory. The beginner in logic should neglect these cross-references.

In the presentation I have tried not to use the 'cabal language' of proof theory but a language familiar to students in mathematical logic.

Since proof theory is a very inhomogeneous area of mathematical logic, a choice had to been made about the parts to be presented here. I have decided to opt for what I consider to be the heart of proof theory - the ordinal analysis of axiom systems. Emphasis is given to the ordinal analysis of the axiom system of the impredicative theory of elementary inductive definitions on the natural numbers. A rough sketch of the 'constructive' consequences of ordinal analysis is given in the epilogue.

Many people helped me to write this book. *J.Columbus* suggested and checked nearly all the exercises. *A.Weiermann* made a lot of valuable suggestions especially in the section about alternative interpretations for Ω. *A.Schlüter* did the proof-reading, drew up the subject index and the index of notations and suggested many corrections especially in the part about the autonomous ordinals of Z_∞.

I am also indebted to the students of the workshop on proof theory in Münster who suggested many more corrections. Last but not least I want to thank all the students attending my course of lectures during the winter term 1987/88. It was their interest in the topic that encouraged me to write this book.

A first version of the typescript was typed by my secretary Mrs. *J.Pröbsting* using the Signum text system. She also wrote the table of contents. Many thanks to all these persons.

July 19, 1989 W. P.
Münster

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