

# Lecture Notes in Mathematics

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B. Simeone (Ed.)

## Combinatorial Optimization

Lectures given at the 3rd Session of the  
Centro Internazionale Matematico Estivo (C.I.M.E.)  
held at Como, Italy, August 25–September 2, 1986

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## PREFACE

The present volume contains the proceedings of the CIME International Summer School on "Combinatorial Optimization", which was held at Villa Olmo, Como, Italy, from August 25 to September 2, 1986.

This was the first CIME Summer School specifically devoted to this quickly developing area, although the Varenna School on "Matroid Theory and its Applications" organized by Prof. Barlotti in 1980 did already include some lectures on matroid optimization. As a matter of fact, the idea of the present School came up for the first time there.

Combinatorial Optimization has a peculiar location in the map of Applied Mathematics, being placed in an interzone in the middle of Combinatorics, Computer Science and Operations Research. From a mathematical point of view, it draws on pure combinatorics, including graphs and matroids, on Boolean algebras and switching functions, partially ordered sets, group theory, linear algebra, convex geometry and probability theory, as well as other tools. Over the past years, a substantial amount of research has been devoted to the connections between Combinatorial Optimization and theoretical Computer Science, and in particular to computational complexity and algorithmic issues. Quite often the study of combinatorial optimization problems is motivated by real-life applications, such as scheduling, assignment, location, distribution, routing, districting, design and other Operations Research applications.

Although references to actual applications were frequently given, the emphasis of the School has been on theoretical aspects of Combinatorial Optimization. The four invited Lecturers, Prof. Peter L. Hammer, Rutgers University, USA; Prof. Ellis L. Johnson, IBM Scientific Research Center, Yorktown Heights, USA; Prof. Bernhard Korte, University of Bonn, West Germany; and Prof. Eugene L. Lawler, University of California, Berkeley, USA, have given a broad account of recent results and current trends in the area. Special attention has been devoted to the study of important classes of functions (either real- or binary-valued) defined on the binary  $n$ -cube (Prof. Hammer); to polyhedral combinatorics and its connections with combinatorial duality theories and min-max identities (Prof. Johnson); to the deep link between

greedy algorithms and finite geometries such as matroids and greedoids (Prof. Korte); to the role of submodularity (a discrete analogue of convexity) and to a general decomposition theory leading to linear-time graph algorithms (Prof. Lawler).

Contributed papers were presented by D. Acketa, C. Arbib, J. Bisschop, S. Dragutin, O. Holland, M. Lucertini, S. Pallottino, G. Pirillo, W. Piotrowski, B. Simeone, and P. Winter, and many of them are collected in this volume. We have also included contributions by M. Conforti and P. Hansen, who had planned to attend the School, but at the last moment were not able to come.

It is a pleasure to acknowledge the financial support of CIME, as well as the valuable assistance provided by Prof. Roberto Conti, Director, and Prof. Antonio Moro, Secretary of CIME. I am grateful to Fondazione "A. Volta" and its Director Prof. Giulio Casati for their kind hospitality: the elegant neo-classic architectures of Villa Olmo and the scenic beauty of Lake Como have created a charming atmosphere for the School; and the local staff, in particular Dr. Chiara De Santis and Mrs. Donatella Marchegiano, has efficiently handled even the tiniest logistic details. I also thank my colleagues Prof. Mario Lucertini and Prof. Stefano Pallottino for their personal help in the organization of the School. Finally, my deepest thanks to the invited Lecturers and to the other participants for their individual contributions to the School.

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