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The Topology of 4-Manifolds



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PREFACE

In the late 1970's, Mike Freedman and I sketched an argument using immersion theory for showing that $\Omega_4^{SO} = Z$. In 1982–83, Iain Aitchison and I worked out new proofs and a reorganization of $\Omega_4^{\text{spin}} = Z$, $p_1 = 3\sigma$, and Rohlin's theorem. In the last 5 years, further simplifications including a yet easier proof of $\Omega_4^{\text{spin}} = Z$ have been found.

A first draft of Chapters XII and XIII was written at IMPA in Rio de Janeiro in fall 1982 and other bits at the University of Maryland in spring 1983, but the bulk of the writing was done at S.-S. Chern's suggestion at the Nankai Institute of Mathematics in May 1987. I was very ably assisted by Bao-zhen Yu, who found some gaps and corrected many errors, not all minor. I am indebted to Charles Livingston and the topology seminar at Indiana who found further gaffes in Fall 1987, and to Berkeley students, particularly Chris Herald, for checking the final version.

Recent work with Larry Taylor on Pin structures and non-orientable generalizations of Rohlin's Theorem has fed back into some further sharpenings of Chapter IV and the proof of Rohlin's Theorem.

Thanks to my collaborators, to IMPA, Maryland, and especially Nankai for their warm hospitality, to Faye Yeager for an excellent TeX manuscript, and to Deb Craig for help with the many figures.

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