Lecture Notes in Mathematics 1793

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Jorge Cortés Monforte

# Geometric, Control and Numerical Aspects of Nonholonomic Systems

Springer

Author

Jorge Cortés Monforte

Systems, Signals and Control Department
Faculty of Mathematical Sciences
University of Twente
P.O. Box 217
7500 AE Enschede
Netherlands

*e-mail: j.cortesmonforte@math.utwente.nl*
*http://www.math.utwente.nl/ssb/cortesmonforte.htm*

Cataloging-in-Publication Data applied for.

Die Deutsche Bibliothek - CIP-Einheitsaufnahme

Cortés Monforte, Jorge:
Geometric, control and numeric aspects of nonholonomic systems / Jorge
Cortés Monforte. - Berlin ; Heidelberg ; New York ; Barcelona ; Hong Kong ;
London ; Milan ; Paris ; Tokyo : Springer, 2002
(Lecture notes in mathematics ; 1793)
ISBN 3-540-44154-9

Cover illustration by María Cortés Monforte

Mathematics Subject Classification (2000): 70F25, 70G45, 37J15, 70Q05, 93B05, 93B29

ISSN 0075-8434
ISBN 3-540-44154-9 Springer-Verlag Berlin Heidelberg New York



Springer-Verlag Berlin Heidelberg New York a member of BertelsmannSpringer
Science + Business Media GmbH

http://www.springer.de



Typesetting: Camera-ready TEX output by the author

SPIN: 10884692 41/3142/ du - 543210 - Printed on acid-free paper

*A mi padre, mi madre, Ima y la Kuka*

# Preface

Nonholonomic systems are a widespread topic in several scientific and commercial domains, including robotics, locomotion and space exploration. This book sheds new light on this interdisciplinary character through the investigation of a variety of aspects coming from different disciplines.

Nonholonomic systems are a special family of the broader class of mechanical systems. Traditionally, the study of mechanical systems has been carried out from two points of view. On the one hand, the area of Classical Mechanics focuses on more theoretically oriented problems such as the role of dynamics, the analysis of symmetry and related subjects (reduction, phases, relative equilibria), integrability, etc. On the other hand, the discipline of Nonlinear Control Theory tries to answer more practically oriented questions such as which points can be reached by the system (accessibility and controllability), how to reach them (motion and trajectory planning), how to find motions that spend the least amount of time or energy (optimal control), how to pursue a desired trajectory (trajectory tracking), how to enforce stable behaviors (point and set stabilization),... Of course, both viewpoints are complementary and mutually interact. For instance, a deeper knowledge of the role of the dynamics can lead to an improvement of the motion capabilities of a given mechanism; or the study of forces and actuators can very well help in the design of less costly devices.

It is the main aim of this book to illustrate the idea that a better understanding of the geometric structures of mechanical systems (specifically to our interests, nonholonomic systems) unveils new and unknown aspects of them, and helps both analysis and design to solve standing problems and identify new challenges. In this way, separate areas of research such as Mechanics, Differential Geometry, Numerical Analysis or Control Theory are brought together in this (intended to be) interdisciplinary study of nonholonomic systems.

Chapter 1 presents an introduction to the book. In Chapter 2 we review the necessary background material from Differential Geometry, with a special emphasis on Lie groups, principal connections, Riemannian geometry and symplectic geometry. Chapter 3 gives a brief account of variational principles in Mechanics, paying special attention to the derivation of the non-

holonomic equations of motion through the Lagrange-d'Alembert principle. It also presents various geometric intrinsic formulations of the equations as well as several examples of nonholonomic systems.

The following three chapters focus on the geometric aspects of nonholonomic systems. Chapter 4 presents the geometric theory of the reduction and reconstruction of nonholonomic systems with symmetry. At this point, we pay special attention to the so-called nonholonomic bracket, which plays a parallel role to that of the Poisson bracket for Hamiltonian systems. The results stated in this chapter are the building block for the discussion in Chapter 5, where the integrability issue is examined for the class of nonholonomic Chaplygin systems. Chapter 6 deals with nonholonomic systems whose constraints may vary from point to point. This turns out in the coexistence of two types of dynamics, the (already known) continuous one, plus a (new) discrete dynamics. The domain of actuation and the behavior of the latter one are carefully analyzed.

Based on recent developments on the geometric integration of Lagrangian and Hamiltonian systems, Chapter 7 deals with the numerical study of nonholonomic systems. We introduce a whole new family of numerical integrators called nonholonomic integrators. Their geometric properties are thoroughly explored and their performance is shown on several examples. Finally, Chapter 8 is devoted to the control of nonholonomic systems. After exposing concepts such as configuration accessibility, configuration controllability and kinematic controllability, we present known and new results on these and other topics such as series expansion and dissipation.

I am most grateful to many people from whom I have learnt not only Geometric Mechanics, but also perseverance and commitment with quality research. I am honored by having had them as my fellow travelers in the development of the research contained in this book. Among all of them, I particularly would like to thank Manuel de León, Frans Cantrijn, Jim Ostrowski, Francesco Bullo, Alberto Ibort, Andrew Lewis and David Martín for many fruitful and amusing conversations. I am also indebted to my family for their encouragement and continued faith in me. Finally, and most of all, I would like to thank Sonia Martínez for the combination of enriching discussions, support and care which have been the ground on which to build this work.

Enschede, July 2002 *Jorge Cortés Monforte*

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