Roland R. Yager · Liping Liu (Eds.)

Classic Works of the Dempster-Shafer Theory of Belief Functions

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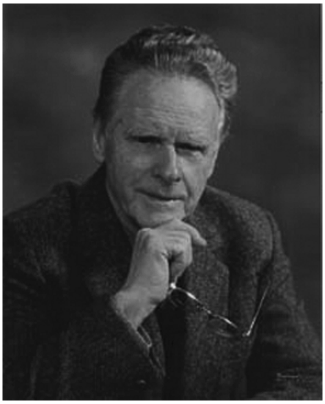
This book is dedicated to the memory of
Philippe Smets (1938–2005),
a lifetime advocate for and contributor to
the Dempster-Shafer Theory of Belief Functions.

About the Founders



Arthur P. Dempster studied mathematics and physics at the University of Toronto, earning a B.A. in 1952 and an M.A. in 1953. He received a Ph.D. in mathematical statistics from Princeton University in 1956. From 1958 through 2005 he served on the active teaching faculty of the Department of Statistics at Harvard University, including 11 years as department chair. He has supervised about 50 Ph.D. theses.

His current title at Harvard is Research Professor of Theoretical Statistics, with interests ranging over the methodology and logic of statistical sciences, technical developments within the Dempster-Shafer framework, and the analysis of biological and physical phenomena. In addition to the theory of belief functions, he is known for the EM algorithm, and a range of contributions to multivariate statistical theory.

He is a fellow of the American Academy of Arts and Sciences, the American Statistical Association, and the Institute of Mathematical Statistics. He was awarded the Kampé de Feriet Award of the International Conference on Information Processing and Management of Uncertainty (IPMU) 2002.

Glenn Shafer is Board of Governors Professor at Rutgers University and a Professor in the Computer Learning Research Centre, Royal Holloway, University of London.

Glenn spent his childhood on a farm near Caney, Kansas. After earning a Ph.D. in mathematical statistics from Princeton in 1973, he taught at Princeton and the University of Kansas, moving to Rutgers in 1992. He and his wife Nell Painter, a distinguished historian, live in Newark, New Jersey, and Glenn serves on the board of the Newark Boys Chorus School.

Glenn's book on the Dempster-Shafer theory, *A Mathematical Theory of Evidence*, appeared in 1976. The most important of his more recent books is *Probability and Finance, It's Only a Game!* (2001, with Vladimir Vovk), which explains how mathematical probability can be based on game theory rather than measure theory. His other major books are *The Art of Causal Conjecture* (1996), concerning the relation between probability and causality, *Probabilistic Expert Systems* (1996), concerning the network structures that facilitate computation for Dempster-Shafer beliefs as well as for conventional probabilities, and *Algorithmic Learning in a Random World* (2005, with Gammerman and Vovk), concerning confidence intervals for machine-learning methods.

Glenn has also published in journals in statistics, philosophy, history, psychology, computer science, economics, engineering, accounting, and law. He was the 2004 recipient of the Daniel Gorenstein Memorial Award for research and service at Rutgers University. He was a Guggenheim fellow in 1983–84, a fellow at the Center for Advanced Study in the Behavioral Sciences in 1988–89, and a Fulbright fellow at the Free University of Berlin in 2001. He is a fellow of the Institute of Mathematical Statistics and the American Association for Artificial Intelligence.

Foreword

This volume is a welcome addition to the literature on the Dempster-Shafer theory. It may help turn the theory, which now enjoys a lively but fragmented existence, into a more coherent and better understood set of tools for probabilistic thinking in science and technology.

The volume's title suggests that the theory had a classical period extending from the 1960s through the 1980s. In its first two decades, it consisted of theoretical writings by the two of us: Dempster's work on upper and lower probabilities in the 1960s and Shafer's work on belief functions in the 1970s. Then interest in applications suddenly flowered. After Jeff Barnett introduced the name "Dempster-Shafer" in 1981 [1], the theory quickly acquired textbook status in artificial intelligence. By the end of the classical period, around 1990, the theory had acquired powerful computational tools, remarkably diverse applications, and the attention of many researchers interested in variations and generalizations.

By many measures, the theory continues to flourish in the 21st century. Internet searches for "Dempster-Shafer" produce ever more hits. The theory is used in many branches of technology, only a few of which are represented in this volume. Articles on the theory and its applications appear in a remarkable number of journals and recurring conferences. Books on the theory continue to appear.

In other important respects, however, the theory has not been moving forward. We still hear questions that were asked in the 1980s: How do we tell if bodies of evidence are independent? What do we do if they are dependent? We still encounter confusion and disagreement about how to interpret the theory. And we still find little acceptance of the theory in mathematical statistics, where it first began 40 years ago.

We have come to believe that three things are needed to move the theory forward.

- **A richer understanding of the uses of probability**. Some authors, including our departed friend Philippe Smets [6], have tried to distance the

Dempster-Shafer theory from the notion of probability. But we have long believed that the theory is best regarded as a way of using probability [2, 4, 5]. Understanding of this point is blocked by superficial but well entrenched dogmas that still need to be overcome.

- **A richer understanding of statistical modeling**. Mathematical statisticians and research workers in many other communities have become accustomed to beginning an analysis by specifying probabilities that are supposed known except for certain parameters. Dempster-Shafer modeling uses a different formal starting point, which may often be equally or more legitimate as a representation of actual knowledge [3].
- **Good examples**. The elementary introductions to the Dempster-Shafer theory that one finds in so many different domains are inadequate guides for dealing with the complications that arise in real problems. We need in-depth examples of sensible Dempster-Shafer analyses of a variety of problems of real scientific and technological importance.

Although neither of us has made the Dempster-Shafer theory our top priority in the last two decades, we plan to address these three challenges in the next few years. We hope that the current volume, by putting earlier contributions to the theory in some order, will encourage others, as it has encouraged us, to take stock of the theory's current state and think about how to address its current challenges.

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Arthur P. Dempster and Glenn Shafer

Preface

This year marks the 40th anniversary of the seminal publication by Arthur P. Dempster on upper and lower probabilities and the 30th anniversary of the classic monograph by Glenn Shafer. These pioneering works established a new theory for probabilistic reasoning based on a generalization of classic probability. Central to this theory is its ability to model imprecision as well as randomness. This capability often makes it superior to Bayesian approaches in modeling knowledge of uncertainty profiles. In the last 30 years, the concept of belief functions has penetrated into many scientific areas and been applied in many projects. The aim of this book is to bring together a collection of classic papers showcasing important theoretical advances and pioneering applications. The book intends to become an authoritative reference for those working in the field of evidential reasoning as well as an important archival reference for those working in a wide range of areas such as information fusion, reasoning under uncertainty, artificial intelligence and decision making in economics, engineering, and management.

The selection of these classic papers was made with the aid of many experts from this field. While the editors did not have a specific definition of what constitutes a classic paper they felt that the following three features should be present in a classic paper. First, the paper should have been published in highly regarded journal, collection, or conference proceedings. Second, the paper must be often recommended by professors to their graduate students as reading materials for research seminars or projects. Finally the paper initiated a stream of research that other scholars have followed leading to an impact on existing research and with a high prospect to continue to make an impact on the future development of the field.

The paper selection process roughly consisted of four stages:

Classic Paper Nominations: The editors elicited nominations through two channels. They first distributed a call for nominations in the academic news lists for Uncertainty in Artificial Intelligence, Operations Research, Association for Information Systems, and American Accounting Association, etc. They then wrote to a dozen prominent scholars in the field to request comments on specific references. They concluded the nomination period with over 100 nominations from over 50 researchers around the world.

Nomination Review: The editors carefully reviewed each nomination to assess its merit. Nominations by senior, well-known scholars were given careful consideration. As a result, the editors created a short list of 69 papers for further review.

Paper Review: The editors reviewed each paper in the short list to assess its overall contribution to the field. This reduces the number of papers under consideration to 40.

Citation Analysis: We made extensive citation analyses to ensure that all papers were influential ones in the field. In addition to eliminating some papers, this process brought two papers not previously nominated into the pool.

Eventually, the editors jointly selected 29 papers, each of which fall into at least one of the following categories: 1) major conceptual innovations that lead to the development of belief functions, establish a mathematical or semantic foundation, build connections with other scientific areas such as fuzzy logic and probabilistic reasoning, and extend the theory of evidence in significant ways; 2) major methodological developments that use belief functions as a tool for scientific research and general problem solving; 3) major computational developments that propose new algorithms or theories, or improve the efficiency of computation involving belief functions; and 4) groundbreaking applications that demonstrate the creative use of belief functions and contribute to the applied area in significant ways.

There is a large body of literature on belief functions and there are many truly outstanding publications that deserve recognition. We could not include all the publications we would like to include, particularly newer ones. Some notable streams of research include those on bridging belief functions to fuzzy logic and imprecise probabilities, those on learning belief functions from data, those on fast or approximate computations, those on belief functions in infinite or continuous frames of discernment, those on adapting Dempster's rule for integrating dependent or highly conflicting sources of information, and those on applying belief functions to a wide range of real problems.

The papers in this volume are in chronological order.

Ronald R. Yager, New York, NY
Liping Liu, Akron, Ohio

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This project was initially suggested by Arthur P. Dempster and received immediate support from Glenn Shafer, Smets Philippe, Jürg Kohlas, and many others. Since it took off in early 2003, it has involved many people in various ways and depths. First, we would like to thank our advisory editors, Art and Glenn, for their clear vision and guidance. Second, we would like to thank Jürg Kohlas and the late Philippe Smets for their careful analysis of the literature. Third, we would like to thank many scholars who participated in the nomination process and provided us with their favorite references. These include Jean-Yves Jaffray, George Klir, Henry Kyburg, Henri Prade, Enrique H. Ruspini, Prakash P. Shenoy, and fifty other distinguished scholars. Fourth, we would like to thank all the authors who have contributed to this volume. Some authors, including Jeffrey A. Barnett, John D. Lowrance, and Philippe Smets, went a long way ahead in providing invaluable help to us by retyping their articles in TeX. In particular, Philippe retyped one of his articles when he was seriously ill. We owe him a giant thank-you and would like to dedicate this volume to his memory. Fifth, we would like to thank Ms. Tong Bin, Ms. Jie Guan, and Mr. Venkata Nuli, who have performed the most tedious and laborious work of the project such as indexing and typesetting. Without them, this project would be impossible. Sixth, we would like to thank Hemalatha, D. and her project team at Integra Software Services Pvt Ltd for the beautiful production of this book.

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Contents