Lecture Notes in Mathematics 1853

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The Valuative Tree

Springer

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To Muriel and Johanna

Preface

This book grew out of a common passion for a beautiful natural object that we decided to call the “valuative tree”. Motivated by questions stemming from complex dynamics and complex analysis, we realized that we needed to understand the link between valuations, which are purely algebraic objects, and more geometric or analytic constructions such as blowups or Lelong numbers. More precisely, we looked at the structure of a special set of valuations, and we found that this space had a very rich and delicate topological structure. We hope that the reader will share our enthusiasm while progressively exploring this space into its finer details all along this book.

This monograph has benefited from the help of many people. The first author wishes to warmly thank Bernard Teissier for his constant support and help, and Patrick Popescu-Pampu, Mark Spivakovsky and Michel Vaquié for fruitful discussions. The second author expresses his gratitude to Jean-François Lafont, Robert Lazarsfeld and Karen Smith. We both thank the referees for a number of useful suggestions.

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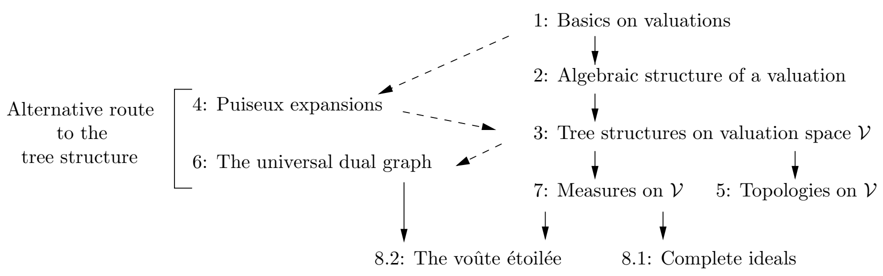
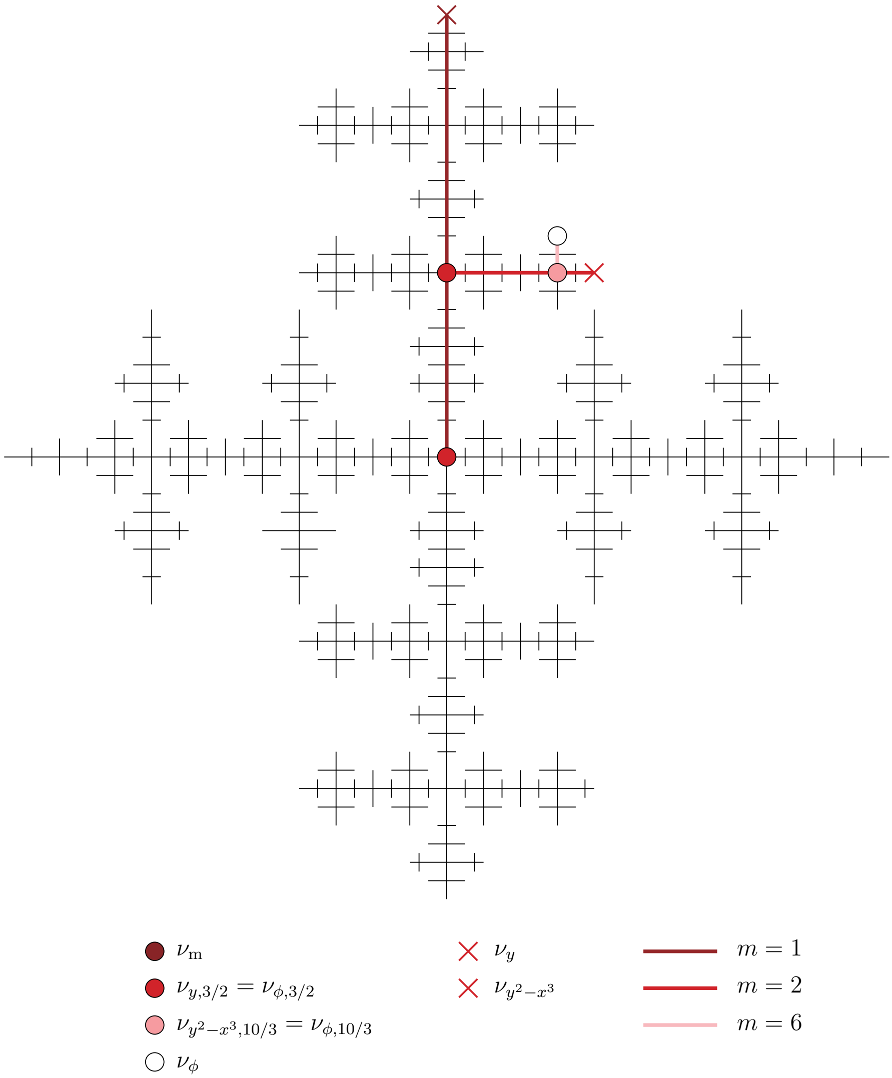
Structure of the Book

Before embarking to a journey into the valuative tree, we describe below the structure of the volume. A plain arrow linking chapter A to chapter B indicates that the understanding of B relies heavily on a previous lecture of A. A dashed arrow indicates a looser link between both chapters.



The Valuative Tree



$$\phi = (y^2 - x^3)^3 - x^{10}$$

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