Lecture Notes in Mathematics

Edited by J.-M. Morel, F. Takens and B. Teissier

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for the publication of monographs

1. Lecture Notes aim to report new developments in all areas of mathematics – quickly, informally and at a high level. Monograph manuscripts should be reasonably self-contained and rounded off. Thus they may, and often will, present not only results of the author but also related work by other people. They may be based on specialized lecture courses. Furthermore, the manuscripts should provide sufficient motivation, examples and applications. This clearly distinguishes Lecture Notes from journal articles or technical reports which normally are very concise. Articles intended for a journal but too long to be accepted by most journals, usually do not have this "lecture notes" character. For similar reasons it is unusual for doctoral theses to be accepted for the Lecture Notes series.

2. Manuscripts should be submitted (preferably in duplicate) either to one of the series editors or to Springer-Verlag, Heidelberg. In general, manuscripts will be sent out to 2 external referees for evaluation. If a decision cannot yet be reached on the basis of the first 2 reports, further referees may be contacted: the author will be informed of this. A final decision to publish can be made only on the basis of the complete manuscript, however a refereeing process leading to a preliminary decision can be based on a pre-final or incomplete manuscript. The strict minimum amount of material that will be considered should include a detailed outline describing the planned contents of each chapter, a bibliography and several sample chapters.
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Authors should also be aware that parallel submission of their manuscript to another publisher while under consideration for LNM will in general lead to immediate rejection.

3. Manuscripts should in general be submitted in English.
Final manuscripts should contain at least 100 pages of mathematical text and should include
- a table of contents;
- an informative introduction, with adequate motivation and perhaps some historical remarks: it should be accessible to a reader not intimately familiar with the topic treated;
- a subject index: as a rule this is genuinely helpful for the reader.

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Lectures on Seiberg-Witten Invariants

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Preface

Riemannian, symplectic and complex geometry are often studied by means of solutions to systems of nonlinear differential equations, such as the equations of geodesics, minimal surfaces, pseudoholomorphic curves and Yang-Mills connections. For studying such equations, a new unified technology has been developed, involving analysis on infinite-dimensional manifolds.

A striking applications of the new technology is Donaldson's theory of "anti-self-dual" connections on $SU(2)$-bundles over four-manifolds, which applies the Yang-Mills equations from mathematical physics to shed light on the relationship between the classification of topological and smooth four-manifolds. This reverses the expected direction of application from topology to differential equations to mathematical physics. Even though the Yang-Mills equations are only mildly nonlinear, a prodigious amount of nonlinear analysis is necessary to fully understand the properties of the space of solutions.

At our present state of knowledge, understanding smooth structures on topological four-manifolds seems to require nonlinear as opposed to linear PDE's. It is therefore quite surprising that there is a set of PDE's which are even less nonlinear than the Yang-Mills equation, but can yield many of the most important results from Donaldson's theory. These are the Seiberg-Witten equations.

These lecture notes stem from a graduate course given at the University of California in Santa Barbara during the spring quarter of 1995. The objective was to make the Seiberg-Witten approach to Donaldson theory accessible to second-year graduate students who had already taken basic courses in differential geometry and algebraic topology.

In the meantime, more advanced expositions of Seiberg-Witten theory have appeared (notably [13] and [32]). It is hoped these notes will prepare the reader to understand the more advanced expositions and the excellent recent research literature.

We wish to thank the participants in the course, as well as Vincent Borrelli, Xianzhe Dai, Guofang Wei and Rick Ye for many helpful discussions on the material presented here.

J. D. MOORE
Santa Barbara
April, 1996

In the second edition, we have corrected several minor errors, and expanded several of the arguments to make them easier to follow. In particular, we included a new section on the Thom form, and provided a more detailed description of the second Stiefel-Whitney class and its relationship to the intersection form for four-manifolds. Even with these changes, the pace is demanding at times and increases throughout the text, particularly in the last chapter. The reader is encouraged to have pencil and paper handy to verify the calculations.

We have treated the Seiberg-Witten equations from the point of view of pure mathematics. The reader interested in the physical origins of the subject is encouraged to consult [9], especially the article, "Dynamics of quantum field theory," by Witten.

Our thanks go to David Bleecker for pointing out that our earlier proof of the Proposition on page 115 was incomplete, and to Lev Vertgeim and an anonymous referee for finding several misprints and minor errors in the text.

J. D. MOORE
Santa Barbara
February, 2001

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