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Spin Glasses

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Preface

Spin glasses have become a paradigm for highly complex disordered systems. In the 1960'ies, certain magnetic alloys were found to have rather anomalous magnetic and thermal properties that seemed to indicate the existence of a new kind of phase transition, clearly distinct from conventional ferromagnetic materials. The origin of these anomalies was soon deemed to lie in two features: the presence of competing signs in the two-body interactions, and the disorder in the positions of the magnetic atoms in the alloy. This has led to the modelling of such materials in the form of spin-systems with random interactions. In the 1970ies, two principle models were proposed: the Edwards-Anderson model, which is a lattice spin system with random nearest neighbor interactions and as such is the randomized version of the classical Ising model; and the Sherrington-Kirkpatrick model, proposed as a mean field model, where all spins interact with each other on equal footing, which is a randomized version of the Curie-Weiss model. The SK-model was clearly intended to provide a simple, solvable caricature of the Edwards-Anderson model, that should give some insights into the nature of the spin glass transitions, just as the Curie-Weiss model allows a partial understanding of ferromagnetic phase transitions. The remarkable interest that the spin glass problem has received is largely due to the fact that neither of the two models turned out to be easily tractable. The Sherrington-Kirkpatrick model was solved on a heuristic level through the remarkable "replica symmetry breaking" ansatz of Parisi, which not only involved rather unconventional mathematical concepts, but also exhibited that the thermodynamic limit of this model should be described by an extraordinarily complex structure. The short-range Edwards Anderson model has been even more elusive, and beyond some rather rudimentary rigorous results, most of our insight into the model is based on numerical simulations, which in themselves prove to be a highly challenging task.

Mathematicians became interested in this problem in the late 1980ies, but on a larger scale in the 1990ies, starting with work of Pastur and Shcherbina, and the systematic programmes initiated by Guerra on the one hand and Talagrand on the other. In 1996 a workshop in Berlin brought together the

leading experts in the field. The state of the art at that time is to a large extend documented in the volume "Mathematical Aspects of Spin Glasses and Neural Networks", edited by A. Bovier and P. Picco (Birkhäuser, 1997). Since then, the progress made in the field has exceeded all expectations. Even as we began planning for a new workshop on the mathematics of spin glasses that was finally held at the Centro Stefano Franscini on the Monte Verità, we did not anticipate that he timing of the event would allow to present for the first time some ground breaking progress. In 2002, Francesco Guerra published an upper bound on the free energy of the SK model that coincided with the Parisi solution. This was the first time that this remarkable construction was to be related to a mathematically rigorous result. Less than a year later, Michel Talagrand announced that he could prove the corresponding lower bound, thus establishing the Parisi solution in a fully rigorous manner.

The Monte Verità meeting thus fell into a most exciting period. It was attended by most of the leading experts on spin glasses, including David Sherrington, Giorgio Parisi, Francesco Guerra, Michel Talagrand, Michael Aizenman, Chuck Newman, and Daniel Stein, to name a few. Besides the reports on the progress mentioned above, the participants and invited speakers reported on a wealth of interesting new results around spin glasses, both on the static and dynamic aspect. As a result we decided to collect a number of invited review papers to document the state of the art in spin glass theory today. The result of this is the present book. It contains a general introduction to the spin glass problem, written by E. Bolthausen, that will serve in particular as a pedagogical guide to the description of the nature of the Parisi solution and the derivation of Guerra's bound in the formulation of Aizenman, Sims, and Star. A. Bovier and I. Kurkova shed light on the Parisi solution from another angle by deriving and describing the asymptotics of the Gibbs measure in another class of spin glass models, the Generalized Random Energy models, in full detail. D. Sherrington gives an account of the history of the spin glass problem from a more physical perspective. M. Talagrand's contribution is a pedagogical presentation of his celebrated proof of the validity of the Parisi solution. Two articles by Ch. Newman and D. Stein discuss the latest developments in the ongoing dispute on the question, whether the predictions of the mean field Sherrington-Kirkpatrick model have any implications for the behavior of short range spin glasses. Finally, A. Guionnet gives an account of what has been achieved in the understanding of another outstanding issue about spin glasses, namely their non-equilibrium properties.

We hope that this volume will serve as a reference handbook for anyone wanting to get an idea of where we are in the theory of spin glasses, and what this subject is all about.

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