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## Harmonic Mappings and Minimal Immersions

Lectures given at the 1st 1984 Session of the  
Centro Internazionale Matematico Estivo  
(C.I.M.E.) held at Montecatini, Italy,  
June 24 – July 3, 1984

Edited by E. Giusti

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## PREFACE

The international course on Harmonic mappings and Minimal Immersions was held at Montecatini, Italy, June 24-July 3, 1984, organized by the Fondazione CIME.

The purpose of the course was to describe recent results and to introduce to the various methods that have been used in the study of harmonic maps between Riemannian manifolds.

The theory of harmonic maps has received several important contributions in recent years, and has been successfully applied in various fields such as minimal surfaces, topology, complex analysis. The course was not aimed at giving a complete survey of all recent advances, but rather at describing in detail some specific topics.

The lecturers, S. Hildebrandt, J. Jost and L. Simon, were given complete freedom in the choice of the subject within the theme of the conference, and have provided an image of a rather broad area. The texts of their lectures form the main body of the present volume.

During the course, there have been two additional lectures by J. Sampson, whose text is included, and by M. Seppala.

I wish to express my gratitude to the lecturers and to all the participants for their contribution to the success of the course.

E. Giusti

C.I.M.E. Session on "Harmonic Mappings and Minimal Immersions"

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