

Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

Subseries: Fondazione C.I.M.E., Firenze

Adviser: Roberto Conti

1159

Schrödinger Operators

Lectures given at the 2nd 1984 Session of the
Centro Internazionale Matematico Estivo
(C.I.M.E.) held at Como, Italy, Aug. 26 – Sept. 4, 1984

Edited by S. Graffi



Springer-Verlag
Berlin Heidelberg New York Tokyo

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Mathematics Subject Classification (1980): 81A, 81C, 81D, 35J, 35P, 35Q, 35R, 35S, 47B, 47E, 47F, 58G

ISBN 3-540-16035-3 Springer-Verlag Berlin Heidelberg New York Tokyo

ISBN 0-387-16035-3 Springer-Verlag New York Heidelberg Berlin Tokyo

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Printing and binding: Beltz Offsetdruck, Hemsbach/Bergstr.
2146/3140-543210

FOREWORD

This volume contains the texts of the lectures and seminars given at the C.I.M.E. Session on Schrödinger Operators, held at the Centro di Cultura Scientifica "A.Volta", Villa Olmo, Como, Italy, from August 27 to September 4, 1984.

The objective of the Session was to provide a broad and up-to-date survey of the meeting-ground common to functional analysis, partial differential equations and quantum mechanics, which goes under the name of Schrödinger operator theory. The three main courses delivered include a thorough coverage of two specific research areas, namely eigenfunctions estimates for the n -body problem by Professor S. Agmon and asymptotic completeness for three-body scattering by Professor V. Enss, and a general overview of basic and recent results by Professor B. Simon. The seminars cover three rapidly expanding research subjects: transition to chaos (J. Bellissard) and probabilistic techniques (G. Jona-Lasinio) in quantum mechanics, and the classical limit by the technique of Fourier integral operators (K. Yajima).

The editor is deeply grateful to his colleagues Professor V. Grecchi and Dr. E. Caliceti and to the staff of the Centro di Cultura Scientifica A.Volta for their assistance in the organization of the Session.

Bologna, March 1985

Sandro Graffi

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