

# Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

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## Integral Representations and Applications

Proceedings of a Conference held at Oberwolfach,  
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Edited by Klaus W. Roggenkamp

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## INTRODUCTION

Integral representation theory and applications thereof have over the past years made considerable progress.

The intention of the Oberwolfach meeting in 1980 was to demonstrate the various applications of orders to number theory, topology, geometry and crystallography and also to show the influences these various subjects have had on integral representation theory. Apart from these, new trends in integral representation theory were presented: Almost split sequences and Auslander-Reiten graphs in connection with classification of indecomposable lattices, and  $\zeta$ -functions on lattices.

Thus we had several survey talks:

"Orders in geometry, topology and number theory";

"Algebraic aspects of crystallography";

"Graham Higman's thesis" (it was surprising to hear of the wealth of results in Higman's 1940 thesis which escaped the attention of people working on the subject - Higman's thesis never being published - and thus were reworked between 1960 and 1980);

"Zeta-functions of orders";

"The class group à la Fröhlich"

"Integral representations in the study of finite Poincaré complexes";

"Poset representations" (unfortunately Ludmilla Nazarova was the only one of the soviet mathematicians working in representation theory who was able to attend the meeting);

"Preprojective lattices over orders over Dedekind domains";

"Projective modules and resolutions for finite groups".

After some hesitation - being aware of the complications - the contributions in these notes appear not alphabetically but according subjects.

Part I contains the historical aspects leading to crystallography and via units to finite conjugate subrings of orders and via personal connections to constructive methods in ideal theory.

Part II deals with  $\zeta$ -functions of orders - influenced by number theory - and Galois module structure - as applications of integral representations to number theory - in its various aspects.

Part III gives applications of integral representation theory to topology, differential geometry and algebraic geometry.

Part IV deals with representation theory eo ipso i.e. classification of indecomposable integral representations, partly in a general setup, partly in concret computations for special group rings and blocks of cyclic defect two.

Part V turns to projective resolutions including relation modules and units in group rings.

Finally I have included in these notes an extended version of two talks, that I gave at "Workshop on Representation Theory of Finite-Dimensional Algebras" in Bielefeld, which presents in a uniform fashion some of the main applications of almost split sequences and Auslander-Reiten quivers.

Since the meeting took place in Oberwolfach there was apart from a tight schedule enough time for a "leisurely" conversation of both personal nature and serious mathematical nature. Hence it is my prime pleasure to express my gratitude to the Forschungsinstitut Oberwolfach. Next I would like to thank the participants for making it a successful meeting. Finally my thanks go to Frau Annemarie Selbor for typing part of the manuscripts and for her secretarial assistance.

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Some of the lectures do not appear in these proceedings, partly since the results will be published elsewhere.

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