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Low Order Cohomology and Applications

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INTRODUCTION

Recently continuous tensor products, infinitely divisible positive definite functions and factorizable representations of "current groups" have received much attention. These apparently very different concepts are connected by the fact that the solution of the relevant problems depends on the knowledge of certain cohomology groups. This was first established by Araki [1] for infinitely divisible positive functions and factorizable representations. The probabilistic aspect of the theory and continuous tensor products were described in [20] and also in [6] .

In chapter I we review the relevant definitions of low order cohomology illustrating them by means of some examples of group extensions.

In chapter II we first give a description of continuous tensor products closely following [26] since this seems to be the most intuitive approach (making use of some facts which have become known since 1968). We also exhibit the connections between first and second order cocycles and continuous tensor products, infinitely divisible projective representations and factorizable projective representations. We give realizations of some of these representations in Fock space.

Chapter III is devoted to the computation of cohomology groups for certain semi-direct products (using the Mackey theory of induced representations). Amongst the examples presented are the Euclidean motion groups and the " Leibniz-Extension" of $SL(2;\mathbb{R})$. These results appear to be new.

The whole of chapter IV is needed for the solution of the cocycle problem for $SL(2;\mathbb{R})$. In order to solve the problem it proved necessary to derive all representations of $SL(2;\mathbb{R})$ as induced representations from <u>one</u> subgroup. Again the explicit computation of the cocycles appears to be new. A result on the dimension of the cohomology group is given in [3], but the proof seems to be incomplete. At the end of the chapter we give the corresponding result for $SL(2;\mathbb{C})$ computed in [7] .

In chapter V we give some powerful theoretical results. These are essentially contained in [32] and [23] . One first needs to prove Kazdan's result and some results on spherical functions in order to see that only the cohomologies of $SU(n;1)$ and $SO(n;1)$ are really of interest. In order to get precise statements about the dimension of these cohomology groups the connection between Lie Algebra and Lie Group is exploited. (Note that the case of SU(1;1) is explicitly excluded here). We just give some indications as to the method of proof and explain the connection with our concrete calculations in chapter IV.

In chapter VI we deal with genuine infinitely divisible representations (thus solving some technical problems). The explicit formulae for these representations are given using the cohomology groups computed earlier.

By means of two examples we show that even representations constructed from non-trivial cocycles are not always irreducible. Here the example of $SO(3) \circledS \mathbb{R}^3$ appears to be new.

In the appendix finally we give the derivation of some results concerning "σ-positive functions" and projective representations which are known but don't seem to be easily accessible in the literature.

It should be mentioned that there appear to be some new results on irreducible representations of current groups (as pointed out by the referee to us). However, at the time of writing, no preprints seem to be available.

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Remark

Most of the results of chapter IV are contained in [4] .