

Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

320

Modular Functions of One Variable I

Proceedings International Summer School
University of Antwerp, RUCA
July 17–August 3, 1972

Edited by W. Kuijk



Springer-Verlag
Berlin Heidelberg New York Tokyo

Editor

Willem Kuijk

**Rijksuniversitair Centrum Antwerpen, Leerstoel Algebra
Groenenborgerlaan 171, 2020 Antwerpen, Belgium**

1st Edition 1973

2nd Corrected Printing 1986

**Mathematics Subject Classification (1970): 10D05, 10D25, 10C15, 14K22,
14K25**

ISBN 3-540-06219-X Springer-Verlag Berlin Heidelberg New York Tokyo

ISBN 0-387-06219-X Springer-Verlag New York Heidelberg Berlin Tokyo

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically those of translation, reprinting, re-use of illustrations, broadcasting, reproduction by photocopying machine or similar means, and storage in data banks. Under § 54 of the German Copyright Law where copies are made for other than private use, a fee is payable to "Verwertungsgesellschaft Wort", Munich.

**© by Springer-Verlag Berlin Heidelberg 1973
Printed in Germany**

**Printing and binding: Beltz Offsetdruck, Hemsbach/Bergstr.
2146/3140-543210**

Preface

An international Summer School on:

"Modular functions of one variable and arithmetical
applications"

took place at RUCA, Antwerp University, from July 17 to August 3, 1972.

This book is the first volume (in a series of four) of the Proceedings of the Summer School. It includes the basic course given by A. Ogg, and several other papers with a strong analytic flavour.

Volume 2 contains the courses of R. P. Langlands (ℓ -adic representations) and P. Deligne (modular schemes - representations of GL_2) and papers on related topics.

Volume 3 is devoted to p -adic properties of modular forms and applications to ℓ -adic representations and zeta functions.

Volume 4 collects various material on elliptic curves, including numerical tables.

The School was a NATO Advanced Study Institute, and the organizers want to thank NATO for its major subvention. Further support, in various forms, was received from IBM Belgium, the Coca-Cola Co. of Belgium, Rank Xerox Belgium, the Fort Food Co. of Belgium, and NSF Washington, D.C.. We extend our warmest thanks to all of them, as well as to RUCA and the local staff (not forgetting hostesses and secretaries!) who did such an excellent job.

P. Deligne
W. Kuyk
G. Poitou
J-P. Serre

CONTENTS

A. OGG,	Survey of Modular Functions of One Variable (Notes by F. Van Oystaeyen)	1
G. SHIMURA,	Complex Multiplication (Notes by Willem Kuyk)	37
G. SHIMURA,	Modular Forms of Half Integral Weight	57
M. ÉICHLER,	The Basis Problem for Modular Forms and the Traces of the Hecke Operators	75
H.M. STARK,	Class-Numbers of Complex Quadratic Fields	153
B. BIRCH,	Some Calculations of Modular Relations	175
J. LEHNER,	On the $A_q(\Gamma) \subset B_q(\Gamma)$ Conjecture	187
Mailing addresses of authors and writers		195