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PREFACE

The past decade has been a period of remarkable activity for complex function theory. An unusual number of open problems, many of long standing, have been settled. At the same time, new techniques of exceptional power continue to be developed. These methods have already yielded a great deal; their promise, if anything, exceeds their past success. An optimist will see in these developments indications of a renaissance of function theory, the achievements of which may ultimately rival the great triumphs of the past.

It was thus especially appropriate for the University of Maryland Mathematics Department to designate the academic year 1973-74 a Special Year in Complex Function Theory. Altogether, well over thirty complex analysts from the United States and abroad participated in a year-long program of seminars, lecture courses, and informal discussions. The present volume is an outgrowth of, and a memorial to, this special year. Partly a (very incomplete) record of material presented at seminars during the year and partly an anthology of results actually obtained during that period, it ranges over a relatively broad expanse of classical and modern function theory: coefficient problems, subharmonic functions, Fuchsian groups, automorphic functions, quasi-conformal mapping, functions of several variables. Uniting these disparate topics is a certain emphasis, in point of view or in method, on problems having concrete geometric content. This seems only natural, for geometric function theory has been the source not only of some of the most difficult and important problems of the general theory, but also of many of its most beautiful and seminal results.

Future volumes, to be published in the Springer Lecture Note Series and the University of Maryland Department of Mathematics Lecture Note Series, will be devoted to material presented in various lecture courses during the special year. For help in editing the

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present volume, we are grateful to Bernard Shiffman and, especially, Leon Greenberg. Thanks also are due Pat Berg and Paula Verdun for an excellent job of typing, and the Mathematics Department of the University of Maryland for having made available the resources required for the preparation of the manuscript.

We hope the reader will find in these papers ample evidence of the continued vigor of the insights of the classical masters and their successors. Complex analysis is indeed alive and well.

W.E. KIRWAN

LAWRENCE ZALCMAN

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