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On the Pointwise Convergence
of Fourier Series



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Dedicated to the memory of my father and mother

Foreword

This monograph is a detailed (essentially) self-contained treatment of the work of Carleson and Hunt and others needed to establish the Main Theorem: If $f \in L^p(-\pi, \pi)$ $1 < p \leq \infty$, then $S_n(x; f)$ converges to $f(x)$ almost everywhere where $S_n(x; f)$ is the n th partial sum of the Fourier series for f .

The purpose of the first five chapters of this monograph is to develop the machinery necessary to reduce the proof of the Main Theorem to the proof (given in Chapter 6) of a theorem that involves only characteristic functions in $L^\infty(-4\pi, 4\pi)$.

In Chapter 6 a large number of statements are presented without proof. Every proof that is omitted in Chapter 6 can be found in Appendix B.

The reader who is not familiar with the concept of Cauchy principal value (denoted: P.V.) or the concept of the Hilbert transform is referred to Appendix A for a summary of the results needed in the sequel.

Appendix C contains the recent results of Kahane and Katznelson on divergence sets which in a sense are opposite to those of Carleson and Hunt.

I would like to thank Professor S.A. Gaal, Professor R.E. Edwards, and Professor S. Kakutani for their encouragement during the preliminary stages of the writing of this monograph, and Professor R.A. Hunt for his very generous help during the spring and summer of 1969 at which time he explained in detail a portion of his original papers to me and made a number of suggestions for improving a few of the proofs contained in them.

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Without his offer of assistance in the spring of 1969 I would not have seriously considered writing this monograph. Also, I would like to thank Professor Y. Katznelson for his permission to reproduce in Appendix C a portion of his text: An Introduction to Harmonic Analysis.

An outline of the proof of Theorem (2.2) based on Theorem (1.18) and Theorem (3.6) was communicated to me by Professor E.M. Stein and my appreciation for his doing so is to be noted here.

This paper was completed while the author was conducting a seminar on the contents of a preliminary draft at Yale University during the fall of 1969. I would like to further acknowledge my indebtedness to the men who participated in this seminar: James Arthur, Eugene J. Boyer, G.I. Gaudry, Michael Keane, S.J. Sidney, Joseph E. Sommese, Charles Stanton, A. Figa-Talamanca and Professors S. Kakutani and C.E. Rickart.

September, 1970

C. J. Mozzochi

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