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Dynamical Systems, Graphs, and Algorithms

 Springer

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The book is dedicated to my three sons — Valeriy, Sergey, Egor
and my wife — Valentina.

Preface

The book presents constructive methods of symbolic dynamics and their applications to the study of continuous and discrete dynamical systems. The main idea is the construction of a directed graph which represents the structure of the state space for the investigated dynamical system. The book contains a sufficient number of examples of concrete dynamical systems from illustrative ones to systems of current interest. Results of their numerical simulations with detailed comments are presented. For an understanding of the book matter, it is sufficient to be acquainted with a general course of ordinary differential equations. The new theoretical results are presented with proofs; the most attention is given to their applications. The book is designed for senior students and researches engaged in applications of the dynamical systems theory.

The base of the presented book is the course of lectures given during the Youth Workshop “Computer Modeling of Dynamical Systems” (June 2004, St. Petersburg) initiated and supported by the UNESCO-ROSTE. Parts of these lectures were presented in ETH, Zurich, 1992; Pohang University of Technology, South Korea, 1993; Belmont University, USA, 1996; St. Petersburg University, Russia, 1999; Suleyman Demirel University, Turkey, 2000; Augsburg University, Germany, 2001; Kalmar University, Sweden, 2004.

Symbolic image, coding, pseudo-orbit, shadowing property, Newton method, attractor, filtration, structural graph, entropy, projective space, Lyapunov exponent, Morse spectrum, hyperbolicity, structural stability, controllability, invariant manifold, chaos.

St. Petersburg – Sebastopol

George Osipenko

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George Osipenko at 1952, Sebastopol, Crimea.

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