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Additive and Cancellative Interacting Particle Systems

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Preface

These notes are based on a course given at the University of Wisconsin in the spring of 1978. The subject is (stochastic) interacting particle systems, or more precisely, certain continuous time Markov processes with state space $S = \{\text{all subsets of } Z^d\}$. This area of probability theory has been quite active over the past ten years: a list of references, by no means comprehensive, may be found at the end of the exposition. In particular, several surveys on related material are already available, among them Spitzer (1971), Dawson (1974b), Spitzer (1974b), Sullivan (1975), Georgii (1976), Liggett (1977) and Stroock (1978). There is rather little overlap between the present treatment and the above articles, and where overlap occurs our approach is somewhat different in spirit.

Specifically, these notes are based on <u>graphical representations</u> of particle systems, an approach due to Harris (1978). The basic idea is to give explicit constructions of the processes under consideration with the aid of <u>percolation substructures</u>. While limited in applicability to those systems which admit such representations, Harris' technique manages to handle a large number of interesting models. When it does apply, the graphical approach has several advantages over alternative methods. First, since the systems are constructed from "exponential alarm clocks," the existence problem does not arise. Also, the uniqueness problem can be handled with much less difficulty than for more general particle systems. Another appealing feature is the geometric nature of the representation, which leads to "visual" probabilistic proofs of many results. Finally, there is the matter of <u>coupling</u>. One of the basic strategies in studying particle systems is to put two or more processes on a joint probability space for comparison purposes. Graphical representations have the property that processes starting from arbitrary initial configurations are <u>all</u> defined on the same probability space, in such a way that natural couplings are often embedded in the construction. This is a major conceptual simplification in many arguments. Altogether, Harris' approach makes the material easily accessible to a gifted graduate student having a familiarity with the elementary theory of Markov chains and processes.

The development is divided into four chapters. Chapter I contains basic notation, general concepts and a discussion of the major problems in the field of interacting particle systems. It also includes a description of the percolation substructures which are used to define the processes we intend to study. Chapter II is devoted to <u>additive systems</u>. The "lineal" additive systems were introduced by Harris (1978). We also consider "extralineal" additive systems. General ergodic and pointwise ergodic theorems are proved. Among the specific models treated in some detail are contact processes, voter models and coalescing random walks. Chapter III deals with <u>cancellative systems</u>, a second large class of models which admit graphical representation. There are analogous general ergodic theorems for this class. Specific topics include an application to the stochastic Ising model, and limit theorems for generalized voter models and annihilating random walks. In Chapter IV we discuss the uniqueness problem for additive and cancellative systems We have closen to present this material last, since uniqueness questions seem rather esoteric in comparison with the important problems of ergodic theory. The graphical approach shows how nonuniqueness can arise when there is "influence from ∞."

A great deal of the material in these notes has appeared in recent research papers by many authors. At the end of each section is a paragraph entitled "Notes" which identifies the sources of the results contained therein. All references are to the Bibliography which follows Chapter IV.

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