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# Lecture Notes in Mathematics

An informal series of special lectures, seminars and reports on mathematical topics

Edited by A. Dold, Heidelberg and B. Eckmann, Zürich

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## Pseudo-Boolean Methods for Bivalent Programming

Lecture at the First European Meeting of the Institute  
of Management Sciences and of the Econometric Institute,  
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## PREFACE

The aim of the present lecture is to propose a method for bivalent  $(0,1)$  linear and nonlinear programming.

We define a pseudo-Boolean function as a real-valued function with bivalent arguments. An equation ( inequality) whose members are pseudo-Boolean functions is named pseudo-Boolean. In this lecture procedures will be proposed for :

- 1) solving systems of linear pseudo-Boolean equations and inequalities;
- 2) solving systems of nonlinear pseudo-Boolean equations and inequalities;
- 3) minimizing a pseudo - Boolean function with or without constraints.

The material of this lecture will form the core of a book on "Boolean Methods in Operations Research and Related Areas", to appear in the "Econometrics and Operations Research" series of the Springer - Verlag.