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The Wulff Crystal in Ising and Percolation Models

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Author

Raphaël Cerf
Mathématiques, Bâtiment 425
UMR CNRS 8628
Université de Paris-Sud
91405 Orsay cedex
France
e-mail: rcerf@math.u-psud.fr

Editor

Jean Picard
Laboratoire de Mathématiques Appliquées
UMR CNRS 6620
Université Blaise Pascal (Clermont-Ferrand)
63177 Aubière Cedex
France
e-mail: jean.picard@math.univ-bpclermont.fr

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To Katti, Benjamin and Nelly

Foreword

Three series of lectures were given at the 34th Probability Summer School in Saint-Flour (July 6–24, 2004), by the Professors Cerf, Lyons and Slade. We have decided to publish these courses separately. This volume contains the course of Professor Cerf. We cordially thank the author for his performance at the summer school, and for the redaction of these notes.

69 participants have attended this school. 35 of them have given a short lecture. The lists of participants and of short lectures are enclosed at the end of the volume.

The Saint-Flour Probability Summer School was founded in 1971. Here are the references of Springer volumes which have been published prior to this one. All numbers refer to the *Lecture Notes in Mathematics* series, except S-50 which refers to volume 50 of the *Lecture Notes in Statistics* series.

1971: vol 307	1980: vol 929	1990: vol 1527	1997: vol 1717
1973: vol 390	1981: vol 976	1991: vol 1541	1998: vol 1738
1974: vol 480	1982: vol 1097	1992: vol 1581	1999: vol 1781
1975: vol 539	1983: vol 1117	1993: vol 1608	2000: vol 1816
1976: vol 598	1984: vol 1180	1994: vol 1648	2001: vol 1837 & 1851
1977: vol 678	1985/86/87: vol 1362 & S-50		2002: vol 1840
1978: vol 774	1988: vol 1427	1995: vol 1690	2003: vol 1869
1979: vol 876	1989: vol 1464	1996: vol 1665	2004: vol 1878 & 1879

Further details can be found on the summer school web site
`http://math.univ-bpclermont.fr/stflour/`

Jean Picard, Université Blaise Pascal
Chairman of the summer school

Preface

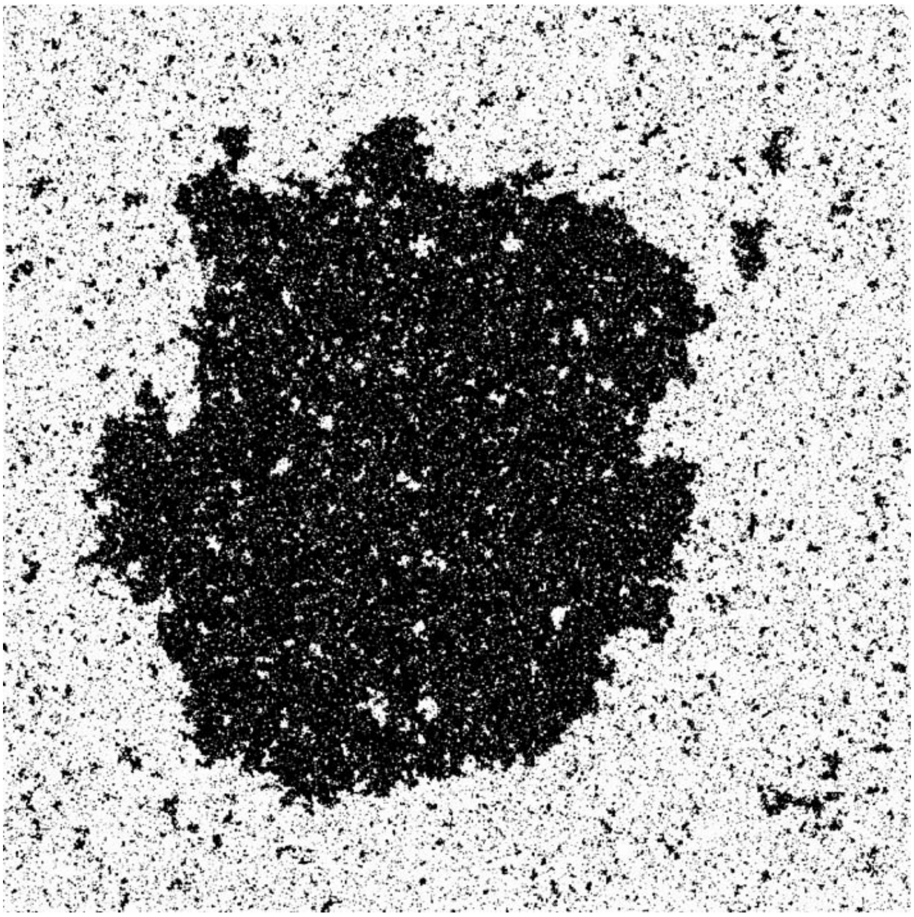
This text is a synthesis of recent works aiming at a mathematically rigorous justification of the phase coexistence phenomenon, starting from a microscopic model. It is intended to be self–contained. The proofs which can be found only in research papers have been included, whereas results for which the proofs can be found in classical textbooks are only quoted.

Here is a brief outline of the structure of the text. The main results on the Wulff crystal in Ising and Percolation models are presented in chapter 5. Throughout the text, I focus on three fundamental models: the Bernoulli percolation model, the FK or random cluster model and the Ising model. These models are respectively introduced in chapters 2, 3, 4. The reader interested mainly in the Wulff crystal of the Ising model can proceed directly to section 5.1. Some background on large deviations is provided in chapter 6, and the surface large deviation principles leading to the results of chapter 5 are stated in chapter 7. The associated volume large deviation principles are presented in chapter 8. Chapters 9, 10, 11, 12 contain the fundamental probabilistic estimates for the proofs. The basic geometric tools involved in the proofs are the object of chapters 13, 14, 15. The final steps of the proofs for each model are described in chapters 16, 17, 18, 19.

The simulations are performed with a one Ghz computer running under Linux. The percolation simulations were done with the program gpbond and the Ising simulations with the program gising. These programs are available under the GNU General Public License through the web page http://www.math.u-psud/˜cerf. The pictures of the droplets of oil in the water were taken in my saucepan.

I thank Jean Picard for the efficient and smooth organization of the Saint–Flour summer school. I wish to express my gratitude to Olivier Couronné, Reda Messikh, Katti Millock and Thierry Quentin de Gromard for their help and comments.

Orsay, Raphaël Cerf
October 2005



Simulation of an Ising Wulff crystal at $T = 2.22$

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Part III Main results

Part IV Large deviation principles

Part V Fundamental probabilistic estimates