

Peter C. Chu  
**P-Vector Inverse Method**

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with 271 Figures and 2 DVDs

 Springer

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## Preface

The demand for oceanic information is increasing rapidly. Physical oceanographers measure the temperature and salinity quite often, but not the velocity. The inverse problem of ocean circulation is to infer from temperature and salinity data, the velocity fields in global oceans and regional seas. The observational temperature and salinity profiles are usually sparse and irregularly distributed. Analysis of the profile data is necessary before using an inverse method. After analyzing  $(T, S)$  profile data, an inverse model on the base of the geostrophic and hydrostatic balances and mass conservation can be used to calculate the absolute velocity  $(u, v, w)$ . The inferred  $(u, v, w)$  data, along with the  $(T, S)$ , provide complete information of physical oceanography for observational study and modeling application.

*P-vector inverse method* covers the complete procedures from the  $(T, S)$  profile data analysis, a simple inverse method (i.e., the P-vector method), including the physical principles and detailed practicing, to wide application of the P-vector inverse method to observational and modeling studies. The emphasis is on the practical application of the subject with many examples. However, the theoretical foundations for various topics are also introduced. The book is intended for graduate or advanced undergraduate students with some basic physical oceanographic knowledge.

The physical oceanographic curricula could use this book for a self-contained course or could be included in courses on ocean data analysis, inverse modeling, ocean analysis and prediction, or ocean circulation. The material is also applicable in marine ecology and related disciplines.

The book is divided into four parts besides the introduction. The first part (Chaps. 2–4) presents the  $(T, S)$  profile data analysis, including the representation of profile data, thermal-haline parametric models, decorrelation scales, establishment of gridded  $(T, S)$  data using the optimal interpolation and optimal spectral decomposition, and various coordinate ( $z$ -, isopycnal, and semi-isopycnal) systems. The second part (Chaps. 5–9) describes theoretical base and technical details of the P-vector inverse method in the  $z$ -, isopycnal-, and semi-isopycnal-coordinate systems. The material covers the

necessary conditions for the velocity inversion, P-vector spiral, evaluation of the P-vector method, variational P-vector method, and global volume transport stream function. In addition, a new unit vector (C-vector) representing the secondary circulation is also included. The third part (Chaps. 10–14) presents the basin/regional scale ocean circulations calculated using the P-vector method. Starting from data format description (NetCDF format) in Chap. 10, circulation in the Atlantic, Pacific, Southern, and Arctic Oceans, and selected regional seas is presented. The fourth part (Chaps. 15, 16) shows the application of the P-vector method in numerical modeling and data assimilation. Use of the calculated absolute velocity along with  $(T, S)$  fields provides the balanced data assimilation and initial condition. When presenting these materials, physical phenomenon and processes (such as barrier layer, seasonal and interannual variability of mixed layer depth, multieddy structures in the regional seas, Kuroshio intrusion into the South China Sea, thermohaline fronts, etc.) and mechanisms are also discussed. In addition, the software for the P-vector inverse method is given in the appendix. The relevant global datasets (NetCDF format) are provided in DVD-ROMs, including  $(T, S)$  atlas in the isopycnal coordinate, heat storage, global, absolute velocity in  $z$ - and isopycnal coordinates, and volume transport streamfunction.

I am grateful to Henry Stommel, Carl Wunsch, Russ Davis, Andrew Bennett, and Peter Killworth for their inspiration on ocean inverse problems. The book could not have been written without the support and cooperation of a number of people. My colleagues, students, and visiting professors/scientists at the Naval Ocean Analysis and Prediction (NOAP) Laboratory at the Naval Postgraduate School helped me in developing the new methods and techniques for observational  $(T, S)$  data analysis and the P-vector concept for ocean circulation inverse: Wenju Cai, Edmo Campos, Michael Carron, Kuofeng Cheng, Jeng-Ming Chen, Yuchun Chen, Nath Edmons, Laura Ehret, Chenwu Fan, Chin-Lung Fang, Charles Fralick, Jose E. Goncalves, Steven D. Heager, Leonid M. Ivanov, Jeffery L. Kerling, Simon Konstantinidis, Kleantih Kyriakidis, Akira Kuninaka, Jian Lan, Ching-Chung Li, Rongfeng Li, W. Timothy Liu, Carlos Lozano, Shihua Lu, Binbing Ma, Tatanya Margolina, Oleg Melnichenko, Gonzalo Montenegro, Rodrigo Obino, Ahchuang Ong, Patrice Pauly, Michael Roth, Carl Szczechowski, Robert Steadley, Hilbert Strauls, Carl Szczechowski, Hsing-Chia Tseng, Joe Veneziano, Guihua Wang, Qianqian Wang, and Susan Wells. Among them, Chenwu Fan's effort is highly appreciated.

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