Lecture Notes in Mathematics 1871

C.I.M.E. means Centro Internazionale Matematico Estivo, that is, International Mathematical Summer Center. Conceived in the early fifties, it was born in 1954 and made welcome by the world mathematical community where it remains, in good health and spirit. Many mathematicians from all over the world have been involved in a way or another in C.I.M.E.'s activities during the past years.

So they already know what the C.I.M.E. is all about. For the benefit of future potential users and co-operators the main purposes and the functioning of the Centre may be summarized as follows: Every year, during the summer, Sessions (three or four as a rule) on different themes from pure and applied mathematics are offered by application to mathematicians from all countries. Each Session is generally based on three or four main courses (24–30 hours over a period of 6-8 working days) held from specialists of international renown, plus a certain number of Seminars.

A C.I.M.E. Session, therefore, is neither a Symposium, nor a School, but maybe a blend of both. The aim is that of bringing to the attention of younger researchers the origins, later developments, and perspectives of some branch of live mathematics.

The topics of the courses are generally of international resonance and the participant of the courses is covering the expertise of different countries and continents. Such combination, gave an excellent opportunity to young participants to be acquainted with the most advanced researches in the topics of the courses and the possibility of an interchange with the world famous specialists. The full immersion atmosphere of the courses and the daily exchange among participants are a first brick in building of international collaboration in Mathematics research.

For more information see CIME's homepage: http://www.cime.unifi.it

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Alexander V. Kazhikov · Yves Meyer · Seiji Ukai

Mathematical Foundation of Turbulent Viscous Flows

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Preface

Over two centuries ago, L. Euler (1750) derived an ideal model equation describing the evolution of fluids. Later on, this model was revised under a more realistic basis by H. Navier (1822) and G. Stokes (1845). Finally, with his eponymous equation L. Boltzmann (1872) introduced the foundation of Gas Dynamics. Since then, much progress has been made in the understanding of these physical models. But many fundamental mathematical questions still remain unresolved, such as the existence, uniqueness and stability of solutions to the corresponding equations in three dimensions.

Due to the large number of applications to different fields (such as meteorology, astrophysics, aeronautics, thermodynamics, lasers and plasma physics), the study of these model equations from a purely mathematical point of view plays a crucial role in Applied Mathematics.

The series of lectures contained in this volume reflects five different and complementary approaches to several fundamental questions arising in the study of the Fluid Mechanics and Gas Dynamics equations. These lectures were presented by five well-known mathematicians at the International CIME Summer School held in Martina Franca, Italy, from 1 to 5 September 2003.

P. Constantin presents the Euler equations of ideal incompressible fluids and discusses the blow-up problem for the Navier-Stokes equations of viscous fluids, also describing some of the major mathematical questions of turbulence theory.

These questions are intimately connected to the Caffarelli-Kohn-Nirenberg theory of singularities for the incompressible Navier-Stokes equations, that is explained in detail in *G. Gallavotti*'s lectures.

A. Kazikhov introduces the reader to the theory of strong approximation of weak limits via the method of averaging, applied to the Navier-Stokes equations.

On the other hand, *Y. Meyer*'s lectures focus on several nonlinear evolution equations – in particular the Navier-Stokes ones – and some related unexpected cancellation properties, that are either imposed on the initial

condition, or satisfied by the solution itself, whenever it is localized in space or in time variable.

Finally, *S. Ukai* presents the asymptotic analysis theory of fluid equations. More precisely, he discusses the Cauchy-Kovalevskaya technique for the Boltzmann-Grad limit of the Newtonian equation, the multi-scale analysis, giving the compressible and incompressible limits of the Boltzmann equation, and the analysis of their initial layers.

Many Ph. D. students and researchers from all over the world attended the summer school, thereby contributing to its success.

The Apulian landscape with its Romanesque and Baroque cathedrals, castles, rocky settlements, trullis and caves, and the city of Martina Franca, with its Ducal Palace – where the lectures were held – contributed to creating an attractive and pleasant working atmosphere. The summer school would not have taken place without the contagious optimism of Vincenzo Vespri, the efficient coordination of Elvira Mascolo and Pietro Zecca and the precious help of Marco Romito and Veronika Sustik. We would like also to thank here Carla Dionisi, who took care of the final typesetting of the lectures notes.

Finally, let us remember that the CIME summer school benefited from the financial support of Ministero degli Affari Esteri – Direzione Generale per la Promozione e la Cooperazione – Ufficio V, M.U.R.S.T., INdAM and Università Franco-Italiana.

July 2004 *Marco Cannone*
Paris, Kanazawa *Tetsuro Miyakawa*

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