Kalman Filtering

Charles K. Chui · Guanrong Chen

Kalman Filtering

with Real-Time Applications

Fifth Edition

Springer

Charles K. Chui
Department of Statistics
Stanford University
Stanford, CA
USA

Guanrong Chen
Department of Electronic Engineering
City University of Hong Kong
Hong Kong
China

ISBN 978-3-319-47610-0 ISBN 978-3-319-47612-4 (eBook)
DOI 10.1007/978-3-319-47612-4

Library of Congress Control Number: 2017933271

Originally published as volume 17 in the series: Springer Series in Information Sciences

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface to the Fifth Edition

This new edition is dedicated to the memory of Rudolf E. Kalman (May 19, 1930–July 2, 2016), with utmost respect and admiration. Kalman was the first to consider the discrete-time Riccati equation for the linear stochastic dynamic model and to derive the optimal linear feedback gain for the corresponding estimator. The importance of this clever innovation is that it enables real-time computations of the optimal estimator, and hence giving birth to the celebrated "Kalman Filter", which is the core of our discussions in this book.

Kalman also initiated the modern systems and control theories by introducing the state-space framework and defining the basic concepts of system controllability and observability, among his many other important contributions to science and technology. To address the latter subject of control systems, we have written another textbook, entitled "Linear Systems and Optimal Control", published in 1989, also by Springer-Verlag, which may be considered as a sister treatise of the present book on Kalman filtering.

In the preparation of this fifth edition, with the assistance from Dr. Wen Yang and Dr. Ling Shi, we have added a new chapter, namely Chap. 12, on the study of distributed estimation of sensor networks. Since this is a focal topic in the current active research on real-time engineering applications of the Kalman filter, we believe it aligns well with the other contents of the book.

We sincerely hope that the readers will find this new edition more comprehensive and informative, and we welcome your generous feedback.

Menlo Park, CA, USA — Charles K. Chui
Hong Kong, China — Guanrong Chen
August 2016

Preface to the Third Edition

Two modern topics in Kalman filtering are new additions to this Third Edition of **Kalman Filtering with Real-Time Applications**. Interval Kalman Filtering (Chap. 10) is added to expand the capability of Kalman filtering to uncertain systems, and Wavelet Kalman Filtering (Chap. 11) is introduced to incorporate efficient techniques from wavelets and splines with Kalman filtering to give more effective computational schemes for treating problems in such areas as signal estimation and signal decomposition. It is hoped that with the addition of these two new chapters, the current edition gives a more complete and up-to-date treatment of Kalman filtering for real-time applications.

College Station and Houston, TX, USA — Charles K. Chui
August 1998 — Guanrong Chen

Preface to the Second Edition

In addition to making a number of minor corrections and updating the list of references, we have expanded the section on "real-time system identification" in Chap. 10 of the first edition into two sections and combined it with Chap. 8. In its place, a very brief introduction to wavelet analysis is included in Chap. 10. Although the pyramid algorithms for wavelet decompositions and reconstructions are quite different from the Kalman filtering algorithms, they can also be applied to time-domain filtering, and it is hoped that splines and wavelets can be incorporated with Kalman filtering in the near future.

College Station and Houston, TX, USA
September 1990

Charles K. Chui
Guanrong Chen

Preface to the First Edition

Kalman filtering is an optimal state estimation process applied to a dynamic system that involves random perturbations. More precisely, the Kalman filter gives a linear, unbiased, and minimum error variance recursive algorithm to optimally estimate the unknown state of a dynamic system from noisy data taken at discrete real-time. It has been widely used in many areas of industrial and government applications such as video and laser tracking systems, satellite navigation, ballistic missile trajectory estimation, radar, and fire control. With the recent development of high-speed computers, the Kalman filter has become more useful even for very complicated real-time applications.

In spite of its importance, the mathematical theory of Kalman filtering and its implications are not well understood even among many applied mathematicians and engineers. In fact, most practitioners are just told what the filtering algorithms are without knowing why they work so well. One of the main objectives of this text is to disclose this mystery by presenting a fairly thorough discussion of its mathematical theory and applications to various elementary real-time problems.

A very elementary derivation of the filtering equations is first presented. By assuming that certain matrices are nonsingular, the advantage of this approach is that the optimality of the Kalman filter can be easily understood. Of course these assumptions can be dropped by using the more well known method of orthogonal projection usually known as the innovations approach. This is done next, again rigorously. This approach is extended first to take care of correlated system and measurement noises, and then colored noise processes. Kalman filtering for nonlinear systems with an application to adaptive system identification is also discussed in this text. In addition, the limiting or steady-state Kalman filtering theory and efficient computational schemes such as the sequential and square-root algorithms are included for real-time application purposes. One such application is the design of a digital tracking filter such as the $\alpha - \beta - \gamma$ and $\alpha - \beta - \gamma - \theta$ trackers. Using the limit of Kalman gains to define the α, β, γ parameters for white noise and the $\alpha, \beta, \gamma, \theta$ values for colored noise processes, it is now possible to characterize this tracking filter as a limiting or steady-state Kalman filter. The state estimation obtained by these much more efficient prediction-correction equations is proved to be near-optimal, in the sense that its error from the optimal estimate decays exponentially with time. Our study of this topic includes a decoupling method that yields the filtering equations for each component of the state vector.

The style of writing in this book is intended to be informal, the mathematical argument throughout elementary and rigorous, and in addition, easily readable by anyone, student or professional, with a minimal knowledge of linear algebra and system theory. In this regard, a preliminary chapter on matrix theory, determinants, probability, and least-squares is included in an attempt to ensure that this text be self-contained. Each chapter contains a variety of exercises for the purpose of illustrating certain related view-points, improving the understanding of the material, or filling in the gaps of some proofs in the text. Answers and hints are given at the end of the text, and a collection of notes and references is included for the reader who might be interested in further study.

This book is designed to serve three purposes. It is written not only for self-study but also for use in a one-quarter or one-semester introductory course on Kalman filtering theory for upper-division undergraduate or first-year graduate applied mathematics or engineering students. In addition, it is hoped that it will become a valuable reference to any industrial or government engineer.

The first author would like to thank the U.S. Army Research Office for continuous support and is especially indebted to Robert Green of the White Sands Missile Range for his encouragement and many stimulating discussions. To his wife, Margaret, he would like to express his appreciation for her understanding and constant support. The second author is very grateful to Prof. Mingjun Chen of Zhongshan University for introducing him to this important research area, and to his wife Qiyun Xian for her patience and encouragement.

Among the colleagues who have made valuable suggestions, the authors would especially like to thank Profs. Andrew Chan (Texas A&M), Thomas Huang (Illinois), and Thomas Kailath (Stanford). Finally, the friendly cooperation and kind assistance from Dr. Helmut Lotsch, Dr. Angela Lahee, and their editorial staff at Springer-Verlag are greatly appreciated.

College Station, TX, USA — Charles K. Chui
January 1987 — Guanrong Chen

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Notations

$A,\ A_k$	System matrices
A^c	"Square-root" of A in Cholesky factorization
A^u	"Square-root" of A in upper triangular decomposition
$B,\ B_k$	Control input matrices
$C,\ C_k$	Measurement matrices
$\mathrm{Cov}(X,\ Y)$	Covariance of random variables X and Y
$E(X)$	Expectation of random variable X
$E(X\|Y = \mathbf{y})$	Conditional expectation
$\mathbf{e}_j, \hat{\mathbf{e}}_j$	
$f(x)$	Probability density function
$f(x_1,\ x_2)$	Joint probability density function
$f(x_1\|x_2)$	Conditional probability density function
$\mathbf{f}_k(\mathbf{x}_k)$	Vector-valued nonlinear functions
G	Limiting Kalman gain matrix
G_k	Kalman gain matrix
$H_k(\mathbf{x}_k)$	Matrix-valued nonlinear function
H^*	
I_n	$n \times n$ Identity matrix
J	Jordan canonical form of a matrix
K_k	
$L(\mathbf{x},\ \mathbf{v})$	
$M_{A\Gamma}$	Controllability matrix
N_{CA}	Observability matrix
$O_{n\times m}$	$n \times m$ Zero matrix
P	Limiting (error) covariance matrix
$P_{k,k}$	Estimate (error) covariance matrix
$P[i,j]$	(i,j)th entry of matrix P
$P(X)$	Probability of random variable X
Q_k	Variance matrix of random vector $\underline{\xi}_k$
R_k	Variance matrix of random vector $\underline{\eta}_k$
$\mathbf{R}^n$	Space of column vectors $\mathbf{x} = [x_1 \cdots x_n]^{\mathrm{T}}$
S_k	Covariance matrix of $\underline{\xi}_k$ and $\underline{\eta}_k$
tr	Trace

$\mathbf{u}_k$	Deterministic control input (at the kth time instant)
$\mathrm{Var}(X)$	Variance of random variable X
$\mathrm{Var}(X\|Y=\mathbf{y})$	Conditional variance
$\mathbf{v}_k$	Observation (or measurement) data (at the kth time instant)
$\mathbf{v}^{2\#}$	
W_k	Weight matrix
$\mathbf{w}_j$	
$(W_\psi f)(b,\ a)$	Integral wavelet transform
$\mathbf{x}_k$	State vector (at the kth time instant)
$\hat{\mathbf{x}}_k, \hat{\mathbf{x}}_{k\|k}$	Optimal filtering estimate of $\mathbf{x}_k$
$\hat{\mathbf{x}}_{k\|k-1}$	Optimal prediction of $\mathbf{x}_k$
$\breve{\mathbf{x}}_k$	Suboptimal estimate of $\mathbf{x}_k$
$\vec{\mathbf{x}}_k$	Near-optimal estimate of $\mathbf{x}_k$
$\mathbf{x}^*$	
$\mathbf{x}^\#,\ \mathbf{x}_k^\#$	
$\\|\mathbf{w}\\|$	"Norm" of $\mathbf{w}$
$\langle \mathbf{x},\ \mathbf{w}\rangle$	"Inner product" of $\mathbf{x}$ and $\mathbf{w}$
$Y(\mathbf{w}_0,\ \cdots,\ \mathbf{w}_r)$	"Linear span" of vectors $\mathbf{w}_0,\ \cdots,\ \mathbf{w}_r$
$\{\mathbf{z}_j\}$	Innovations sequence of data
$\alpha,\ \beta,\ \gamma,\ \theta$	Tracker parameters
$\{\underline{\beta}_k\},\ \{\underline{\gamma}_k\}$	White noise sequences
$\Gamma,\ \Gamma_k$	System noise matrices
δ_{ij}	Kronecker delta
$\underline{\epsilon}_{k,\ell},\ \underline{\overline{\epsilon}}_{k,\ell}$	Random (noise) vectors
$\underline{\eta}_k$	Measurement noise (at the kth time instant)
$\underline{\xi}_k$	System noise (at the kth time instant)
$\Phi_{k\ell}$	Transition matrix
df/dA	Jacobian matrix
$\partial\mathbf{h}/\partial\mathbf{x}$	Jacobian matrix