

# The Lidov-Kozai Effect – Applications in Exoplanet Research and Dynamical Astronomy

# Astrophysics and Space Science Library

---

## EDITORIAL BOARD

### *Chairman*

W. B. BURTON, *National Radio Astronomy Observatory, Charlottesville, Virginia, U.S.A. (bburton@nrao.edu); University of Leiden, The Netherlands (burton@strw.leidenuniv.nl)*

F. BERTOLA, *University of Padua, Italy*

C. J. CESARSKY, *Commission for Atomic Energy, Saclay, France*

P. EHRENFREUND, *Leiden University, The Netherlands*

O. ENGVOLD, *University of Oslo, Norway*

A. HECK, *Strasbourg Astronomical Observatory, France*

E. P. J. VAN DEN HEUVEL, *University of Amsterdam, The Netherlands*

V. M. KASPI, *McGill University, Montreal, Canada*

J. M. E. KUIJPERS, *University of Nijmegen, The Netherlands*

H. VAN DER LAAN, *University of Utrecht, The Netherlands*

P. G. MURDIN, *Institute of Astronomy, Cambridge, UK*

B. V. SOMOV, *Astronomical Institute, Moscow State University, Russia*

R. A. SUNYAEV, *Space Research Institute, Moscow, Russia*

More information about this series at <http://www.springer.com/series/5664>

Ivan I. Shevchenko

# The Lidov-Kozai Effect – Applications in Exoplanet Research and Dynamical Astronomy



Springer

Ivan I. Shevchenko  
Pulkovo Observatory of the Russian  
Academy of Sciences  
St. Petersburg, Russia

ISSN 0067-0057                      ISSN 2214-7985 (electronic)  
Astrophysics and Space Science Library  
ISBN 978-3-319-43520-6              ISBN 978-3-319-43522-0 (eBook)  
DOI 10.1007/978-3-319-43522-0

Library of Congress Control Number: 2016951301

© Springer International Publishing Switzerland 2017

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Cover illustration: An exoplanet seen from its moon (artist's impression). Credit: IAU/L. Calçada

Printed on acid-free paper

This Springer imprint is published by Springer Nature  
The registered company is Springer International Publishing AG Switzerland

# Preface

Up to now, it nearly makes me laugh and cry of rapture when I recall this result.

Vladimir Beletsky, *Six Dozens (Memoirs)*

The Lidov-Kozai effect (LKE), as any other basic astrophysical phenomena, has several “faces”, or manifestations. The most familiar face, granted in Wikipedia, is the phenomenon of coupled periodic variations (which can be very large) of the inclination and eccentricity of an orbiting body, which (the variations) may take place in the presence of an inclined-enough perturber.

A face, less familiar to the general audience, but physically the generic one, is the so-called phenomenon of  $\omega$ -libration; i.e., libration of the *argument of pericenter* of an orbiting body, when an inclined-enough perturber is present. In fact, the  $\omega$ -libration is at the core of the LKE: it is just a large-amplitude  $\omega$ -libration that entails the mentioned coupled variations in inclination and eccentricity.

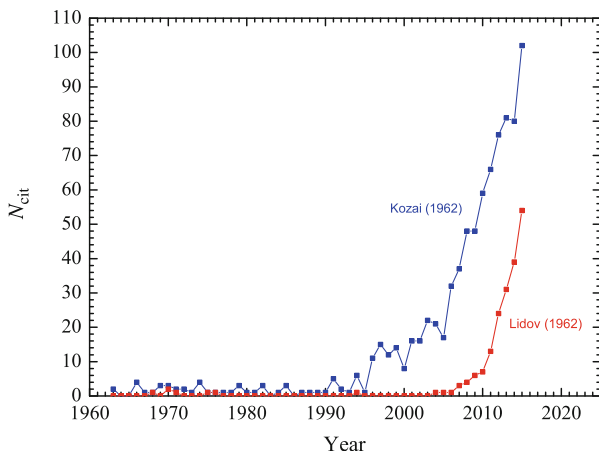
Another concept, frequently present in the modern astrophysical literature, is the LKE as a valuable tool to explain various merger events. Indeed, if it is “necessary to merge” any gravitating binary (say a binary consisting of two ordinary stars, or black holes, or asteroids, or a star and a “hot Jupiter,” or a planet and an artificial satellite, etc.) into a single object, the LKE is commonly the first one in the queue of possible explaining mechanisms that come to mind of an astrophysicist. This is just because it is rapid and no dissipation is needed—only the presence of a perturber. This third face revealed itself already in the pioneering works of Lidov (who outlined a condition for a merger of a satellite with the Earth, in particular the Moon with the Earth), and since then it has been becoming only more and more attractive for researchers.

To date, the LKE, in all of these faces, has been verified to be important in the dynamics of a lot of kinds of astrophysical objects. Historically the applications started with satellites and asteroids; now they comprise comets, trans-Neptunian objects, exoplanets, multiple stars, and many others. Perhaps, of most interest for the astronomical and astrophysical community at present is its relevance for many exoplanetary systems.

Recent years witnessed major advancements in the theory of LKE. In particular, another face of the LKE emerged: the so-called *flip* (orbit turnover) phenomenon.

It would be no exaggeration to say that at present the Lidov-Kozai effect becomes one of the most studied astrophysical effects; this is manifested, in particular, in a

sharp rise of citations of the pioneer articles of Lidov and Kozai during recent years (see Fig. 1).



**Fig. 1** The per-year number of citations of the pioneer articles of Lidov and Kozai, as a function of time, up to year 2015 inclusive (The citation data of the NASA ADS database have been used to construct the diagram)

The topics, covered in this book, include: historical background for appearance of works by Lidov and Kozai in the beginning of the 1960s (*Luna-3*, etc.); modern secular theories and integrable approximations in celestial mechanics; an overview of classical works on the Lidov-Kozai effect; modern advancements and generalizations in the Lidov-Kozai theory, in particular in the framework of noncircular and nonrestricted problems; the Lidov-Kozai mechanism explaining the observed orbital configurations of irregular satellites, explaining comets-sungrazers, and explaining the observed dynamical patterns in the asteroid and Kuiper belts; the role of the Lidov-Kozai mechanism (in particular the Lidov-Kozai migration) in sculpting exoplanetary systems; applications in stellar dynamics, such as scenarios for formation of close binary stars; and explanations for highly eccentric stellar orbits in the Galactic center.

The initial aim of this book was to provide the most full coverage of the effect's theory and applications. However, due to a large amount of the published and newly appearing advancements, it is practically impossible to fulfill this aim in detail. Therefore, apart from the provided bibliography, for further reading on the subject, I would recommend the material on the Lidov-Kozai effect in asteroidal dynamics, presented in Sects. 8.2 and 11.2.2 of the book *Modern Celestial Mechanics* by A. Morbidelli (2002), and the material on the LKE in exoplanetary dynamics presented in Sects. 7 and 8 of the review "The Long-Term Dynamical Evolution of Planetary Systems" by M. B. Davies et al. (2014).

The presented book is self-contained: only basic knowledge in mathematics and mechanics is required for understanding the material, if read from the beginning. I hope that the book can be helpful for a researcher working in astrophysics, celestial mechanics, stellar dynamics, theoretical mechanics, and space mission design, at any level (researcher, graduate student, undergraduate student), depending on the interests of the reader.

I am most grateful to Vladislav Sidorenko, Konstantin Kholshchevnikov, Mikhail Vashkovyakov, and Alessandro Morbidelli for valuable remarks and comments. I am especially thankful to Vladislav Sidorenko for providing rare bibliographic materials on the subject.

Saint Petersburg, Russia  
2016

Ivan I. Shevchenko





# Contents

<b>1</b>	<b>Dynamical Essence and Historical Background</b> .....	1
1.1	The Keplerian Orbital Elements .....	2
1.2	The “omega-libration” .....	5
1.3	The Breakthrough Premises .....	6
1.4	From <i>Luna-3</i> to Modern Space Missions .....	9
<b>2</b>	<b>Averaging and Normalization in Celestial Mechanics</b> .....	13
2.1	The Hamiltonian Formalism .....	14
2.2	The Two-Body Problem in a Hamiltonian Form .....	15
2.3	An $N$ -Body Problem in a Hamiltonian Form .....	16
2.4	The Delaunay Variables .....	17
2.5	Near-Integrable Systems .....	19
2.6	The von Zeipel Method .....	20
2.7	The Hori–Deprit Method .....	23
<b>3</b>	<b>Classical Results</b> .....	27
3.1	A Single-Averaged R3BP .....	28
3.2	The Double-Averaged R3BP .....	31
3.2.1	The Lidov–Kozai Hamiltonian .....	31
3.2.2	Equations and Constants of Motion .....	34
3.2.3	Classification of Orbits .....	36
3.2.4	The Lidov–Kozai Diagrams .....	39
3.2.5	The Solution in the Jacobi Elliptic Functions .....	41
3.3	LKE-Preventing Phenomena .....	46
3.3.1	Perturbations by Additional Orbiting Bodies .....	47
3.3.2	Primary’s Oblateness .....	48
3.3.3	Tides .....	50
3.3.4	General Relativity .....	52
3.3.5	The Orbital Precession in Total .....	53
3.4	Critical Radii .....	54

- 4 The Theory Advances** ..... 57
  - 4.1 LKE in the Non-hierarchical Circular R3BP ..... 58
  - 4.2 LKE in Presence of Mean Motion Resonances ..... 60
  - 4.3 The “eccentric LK-mechanism” ..... 61
  - 4.4 The Stellar Three-Body Problem in Octupole Approximation ..... 62
    - 4.4.1 Triple Stars in the Galaxy ..... 63
    - 4.4.2 The Hierarchical Stellar Three-Body Problem in Historical Perspective ..... 63
    - 4.4.3 Equations of Motion in the Jacobi Frame ..... 65
    - 4.4.4 The Octupole Hamiltonian of the Stellar Problem ..... 67
    - 4.4.5 Technical Dangers of Formal Elimination of Nodes ..... 72
    - 4.4.6 Octupole Approximation Versus Quadrupole Approximation: New Behaviours ..... 74
  - 4.5 Timescales of the LKE ..... 76
    - 4.5.1 Timescales of the Classical LK-Oscillations ..... 76
    - 4.5.2 Timescales of the Eccentric LK-Mechanism ..... 79
  - 4.6 LKE: Resonance or Not ..... 83
    - 4.6.1 Nonlinear Resonance in the Pendulum Model ..... 83
    - 4.6.2 The Place of LK-Resonance in the General Typology of Resonances ..... 87
- 5 Understanding Irregular Satellites** ..... 91
  - 5.1 Irregular Satellites: Origin and Orbits ..... 94
    - 5.1.1 Where They Came From ..... 94
    - 5.1.2 Orbital Distributions ..... 95
  - 5.2 Jovian System ..... 96
  - 5.3 Saturnian System ..... 99
  - 5.4 Uranian and Neptunian Systems ..... 101
- 6 Sungrazing Comets** ..... 105
  - 6.1 Cometary Dynamics Subject to LKE ..... 106
    - 6.1.1 The Jacobi Integral and Tisserand Relation ..... 106
    - 6.1.2 Comets in Highly Inclined Orbits ..... 111
  - 6.2 Origin of Sungrazers ..... 113
  - 6.3 The Oort Cloud and Cometary Transport ..... 114
- 7 Asteroids and Kuiper Belt Objects in Inclined Orbits** ..... 117
  - 7.1 The Kozai Hamiltonian and Diagrams ..... 118
  - 7.2 Inclined Asteroids: Inside Perturber’s Orbit ..... 122
  - 7.3 Inclined TNOs: Outside Perturber’s Orbit ..... 127
  - 7.4 Inclined Asteroids and TNOs in Mean Motion Resonances ..... 132
    - 7.4.1 Mean Motion Resonances ..... 132
    - 7.4.2 Secular Resonances ..... 133
  - 7.5 A Resonant “Dance” of 2335 James ..... 136

<b>8 The Role in Sculpting Exoplanetary Systems</b> .....	139
8.1 Secular Dynamics of Exoplanetary Systems .....	140
8.2 LKE in Multiplanet Systems .....	143
8.3 LKE in Planetary Systems of Binary Stars .....	145
8.4 The Lidov-Kozai Migration and the Origin of “hot Jupiters” .....	148
8.5 Producing Retrograde Orbits? .....	151
8.6 LKE and Dynamical Chaos .....	156
8.6.1 Dynamical Chaos Due to Resonance Overlap .....	156
8.6.2 Chaos in the Planetary Motion Subject to LKE .....	158
<b>9 Applications in Stellar Dynamics</b> .....	161
9.1 LKE in Triples: Formation of Tight Binaries .....	162
9.2 LKE in Triples: A Progenitor for Supernovae, “blue stragglers”, etc. ....	164
9.3 Highly Eccentric Stars in the Galactic Center .....	166
9.4 The Galactic Tide .....	168
<b>A Basic Notations</b> .....	171
<b>B Astronomical Constants and Parameters</b> .....	175
<b>References</b> .....	177
<b>Index</b> .....	189