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Many mathematicians from all over the world have been involved in a way or another in C.I.M.E.'s activities over the years. The main purpose and mode of functioning of the Centre may be summarised as follows: every year, during the summer, sessions on different themes from pure and applied mathematics are offered by application to mathematicians from all countries. A Session is generally based on three or four main courses given by specialists of international renown, plus a certain number of seminars, and is held in an attractive rural location in Italy.

The aim of a C.I.M.E. session is to bring to the attention of younger researchers the origins, development, and perspectives of some very active branch of mathematical research. The topics of the courses are generally of international resonance. The full immersion atmosphere of the courses and the daily exchange among participants are thus an initiation to international collaboration in mathematical research.

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Annalisa Buffa • Giancarlo Sangalli

Editors

IsoGeometric Analysis: A New Paradigm in the Numerical Approximation of PDEs

Cetraro, Italy 2012



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Preface

Isogeometric analysis (IGA) refers to a collection of methods, first introduced by T.J.R. Hughes and collaborators in the seminal paper [3], that use splines, or some of their generalisations such as NURBS (non-uniform rational B-splines), T-splines and hierarchical splines, as functions to build approximation spaces which are then used to numerically solve partial differential equations (PDEs). Indeed, splines and their extensions are the basic mathematical engine behind CAD systems, and one of the main motivation for IGA was to design numerical methods able to avoid remeshing steps by using the output of CAD systems directly. In the last decade, isogeometric methods have attracted a great interest of the scientific community, as it is clearly documented by the number of publications on the subject that appeared in the fields of geometric modelling, mechanical engineering and numerical analysis of PDEs.

Isogeometric methods have been tested and applied on a variety of problems of engineering interest, and there is indeed a large engineering literature showing the beneficial effects of higher regularity in several practical problems; see, for instance, the references given in [2] and in the introduction of Beirão da Veiga et al. [1]. Indeed, the large activity around this class of methods shows that splines yield a powerful approach to PDEs discretisation and that, thanks to the regularity of trial and test spaces, spline-based methods enjoy features that would be hard to achieve with classical finite elements. Examples are the design of regular and fully conservative discretisations of flow and electromagnetic problems or the natural discretisations of high-order PDEs.

The aim of this C.I.M.E. summer school has been to give an introduction to isogeometric methods with a focus on their mathematical foundations. The four chapters of this book collect the main topics that have been presented during the lectures.

Chapter “Standard and Non-standard CAGD Tools for Isogeometric Analysis: A Tutorial” authored by Carla Manni and Hendrik Speleers, is a tutorial on splines and generalisations that are used in CAD parametrisations and form the building blocks of isogeometric methods. Chapter “Models for Isogeometric Analysis from CAD”, by Vibeke Skytt and Tor Dokken, gives an overview of geometric modelling

techniques that can be used with isogeometric methods. Chapter “An Introduction to the Numerical Analysis of Isogeometric Methods”, written by Lourenco Beirão da Veiga, Rafael Vázquez and ourselves, is an overview of mathematical properties of isogeometric spaces, and finally in chapter “Isogeometric Compatible Discretizations for Viscous Incompressible Flow”, by John A. Evans and Thomas J.R Hughes, gives an example application of fundamental importance, the isogeometric simulation of a viscous incompressible flow.

We express our deepest gratitude to all the people who have contributed to the success of this C.I.M.E. summer school: the lecturers, the speakers, the authors who have contributed to this *C.I.M.E. Foundation Subseries* book and the participants. We thank Federico Marini for his help in the preparation of this volume. Last but not least, we thank C.I.M.E., in particular Elvira Mascolo (current C.I.M.E. Director) and Pietro Zecca (former C.I.M.E. Director) for their continuous support in the organisation of the school.

Pavia, Italy
Pavia, Italy
January 2016

Annalisa Buffa
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