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Local Features in Natural Images via Singularity Theory

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To our families
Joanne and Jimi
Rachel, Stephen and Charlotte
Mike and Sandy

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